

# A Statistical Analysis of Luck

Isaac Wilhelm

Rutgers University

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# Outline of Talk

1. Pritchard's analysis of luck, and its problems.
2. The statistical analysis of luck.
3. Evaluating the statistical analysis.

## Pritchard's Analysis of Luck (PAL)

Event  $E$  is lucky if and only if

1.  $E$  obtains in the actual world, and
2. there is a sufficiently large class of possible worlds at which  $E$  fails to obtain, which are all
  - (i) sufficiently close to the actual world, and
  - (ii) sufficiently similar to the actual world in the sense that the relevant initial conditions for  $E$  are the same (Pritchard, 2014).

## Attractions of the PAL: Intuitive Cases

The PAL classifies many intuitively lucky events as lucky, and classifies many intuitively non-lucky events as non-lucky.

### Millie's Near-Accident

- A car speeds around a corner, missing Millie by an inch. Let  $E$  be the event of Millie's near-accident.
  - $E$  is lucky: in many close possible worlds, Millie gets hit.
    - Perhaps the driver turns the wheel a tenth of a degree further.

# Attractions of the PAL: Intuitive Cases

## RELIABLE NEWSPAPER

- Emily and Stephen each buy a lottery ticket.
  1. Emily forms the true belief that her ticket is a loser ( $b_1$ ) by reading the winning number in a reliable newspaper.
    - $E_1$ : the event of Emily's belief formation.
  2. Stephen forms the true belief that his ticket is a loser by reasoning from probabilities.
    - $E_2$ : the event of Stephen's belief formation.
- $E_1$  is not lucky: in all modally close worlds, the reliable newspaper does not make a mistake.
- $E_2$  is lucky: in a cluster of close possible worlds, Stephen wins because the lottery balls jostle slightly differently.

## Attractions of the PAL: Gradability

Luck comes in degrees.

- Millie's near-accident is luckier than Stephen's formation of a true belief.

According to the PAL, luck is indeed gradable.

- The notions of 'sufficiently large' classes of worlds, 'sufficiently close' worlds, and 'relevant' initial conditions all admit of degrees.

## Attractions of the PAL: Objectivity

The PAL treats luck as an objective feature of events, determined by objective similarity metrics among possible worlds.

- According to the PAL, luck is mind-independent and language-independent.
- Facts about luck are independent of luck attributions.
  - Though luck attributions tend to track luck facts.

# BURIED TREASURE

1. Glory buries treasure in the northwest part of her island, under the only patch of land that can support rose bushes.
  - Her desire to bury the treasure in the northwest, in a place that supports roses, is a deep feature of her psychology.
2. William goes to plant roses in the northwest part of the island.
  - His desire to plant roses in the northwest is a deep feature of his psychology.
3. William discovers the treasure (Lackey, 2008).



# BURIED TREASURE

- Intuitively, William's discovery is lucky.
  1. Between the burial and the discovery, many events might have prevented William from finding the treasure.
  2. William's preferences and Glory's preferences might not have converged so neatly.
- According to the PAL, William's discovery is non-lucky.
  - In all sufficiently close worlds, he discovers the treasure.

## RELIABLE NEWSPAPER

- McEvoy (2009) argues that according to the PAL,  $E_1$  is not lucky.
  - There is a cluster of sufficiently close worlds in which Emily wins, but comes to believe that  $b_1$  because of a misprint.
    - A fly gets squashed in the printing apparatus.
    - So the wrong number is printed on the paper Emily receives.

$b_1$ : the belief that Emily's ticket is a loser.

$E_1$ : the event of Emily forming the belief that  $b_1$ .

# RELIABLE NEWSPAPER

- How to adjudicate between McEvoy's reading of RELIABLE NEWSPAPER and Pritchard's reading?
- The theoretical notions invoked by the PAL are not precise enough to decide between them.
  - The imprecise notions: 'sufficiently large' classes of worlds, 'sufficiently close' worlds, and 'relevant' initial conditions.
- So according to the PAL, there is not a determinant fact of the matter as to whether  $E_1$  is lucky to a particular degree.
  - This makes it hard to see how luck can be an objective feature of events.

## Changing the PAL

- To remedy the issue raised by BURIED TREASURE, simply drop the stipulation that the class of worlds at which a lucky event fails must be modally close.
- To remedy the issue raised by RELIABLE NEWSPAPER, precisify the notion of a 'sufficiently large' class of worlds, and the notion of a 'relevant' initial condition.

# First-Pass Characterization of Luck

Event  $E$  is *lucky* just in case it actually occurs, but it fails to occur in a sufficiently large fraction of modal space in which the relevant initial conditions are the same as in the actual world.

Relevant initial conditions should include:

- our world's physical laws;
- our world's macrostate.

# The Mentaculus

The first-pass characterization can be precisified via the Mentaculus.

- The Mentaculus consists of one contingent empirical fact and three laws (Albert, 2000; Loewer, 2016).
  - The contingent fact: the macrostate of the entire universe.
  - The three laws:
    1. Dynamical Law (DL);
    2. Past Hypothesis (PH);
    3. Statistical Postulate (SP).

# The Universe's Macrostate ( $M$ )

- $M$ : the directly surveyable condition of the universe.
- $M$  consists of all macrofacts about the universe at a particular time.
- It corresponds to a macroregion  $R_M$  of the universe's phase space.
  - Points in phase space represent arrangements of particles in the universe.

## Dynamical Law (DL)

- DL: the laws of temporal evolution are Newtonian.
- This evolution is described by possible trajectories ('DL-trajectories') that microconditions may take through the universe's phase space.
- Just as the universe exhibits different macrostates over time, so the DL-trajectory of the universe's phase space point passes through different regions of phase space.



## Past Hypothesis (PH)

- PH: the initial macrostate of the universe was one of extremely low entropy.
- Entropy: change in heat per unit temperature.
- If PH were excluded from the relevant initial conditions, the forthcoming analysis of luck would be significantly distorted.

## Statistical Postulate (SP)

All points in phase space compatible with the actual macrostate (and with  $R_{PH}$ ) are 'equally likely' to be the actual microstate.

- Take all the points in phase space compatible with the actual macrostate of the universe. This is just  $R_M$ .
- Consider only those points in  $R_M$  which came from  $R_{PH}$  by following DL-trajectories.
- According to SP, each of those points is equally likely to be the one which represents the actual microstate.

## The Probabilities of SP

- SP may be used to specify the probability that a given macroevent  $E$  will occur.
- Let  $Pr$  be the 'equally likely' probability distribution.
- The probability of  $E$  is the probability that the universe's phase space point lies on a DL-trajectory that begins in  $R_{PH}$ , passes through  $R_M$ , and eventually arrives at  $R_E$ .
- Symbolically:  $Pr(E \mid M \ \& \ DL \ \& \ PH)$ .

# The SAL

Event  $E$  is lucky to degree  $d$  (relative to macrostate  $M$  at time  $t$ ) if and only if

1.  $E$  occurs in the actual world, and
2.  $Pr(E \mid M \ \& \ DL \ \& \ PH) = 1 - d$ .

# Comments on the SAL

1. According to the SAL, the luckiness of an event is always relative to the macrostate of the universe at a particular time.
  - Seems good. William's discovery is not lucky, relative to the universe's macrostate one millisecond before his shovel hits the chest.
2. Context determines:
  - which macrostate is invoked in luck attributions;
  - what the threshold for being lucky *simpliciter* is.
3. The SAL is simply a more precise version of the first-pass characterization.

## Comments on the SAL

1. The SAL is quite similar to the PAL.
  - Both are modal.
  - Both take lucky events to be those that happen infrequently across a range of possible worlds.
2. The difference between the SAL and the PAL concerns which worlds are included in that range.
  - PAL: only close possible worlds.
  - SAL: all physically possible worlds compatible with the universe's macrostate.

# Attractions

The SAL has the three attractive features that the PAL has:

1. gets intuitive cases right;
2. accounts for the gradability of luck;
3. treats luck as objective.

## SAL and BURIED TREASURE

William's discovery is lucky, according to the SAL.

1. Let  $P$  be a macrostate that obtains before Glory and William form their preferences.
2. Relative to  $P$ , the probability that William does not find treasure is very high.
  - In some microconditions, William comes to love the northeast.
  - In others, Glory comes to love canna lilies rather than roses.
3. So according to the SAL, the discovery is lucky to an extremely high degree.
4. Since the contextually-determined threshold is set by significance, it is likely to be low in this case.
5. Therefore, the discovery is lucky simpliciter.



## SAL and BURIED TREASURE

William's discovery is also lucky relative to other macrostates.

1. Let  $P'$  be the macrostate that occurs after Glory buries her treasure, but long before William discovers it.
2. Again, relative to  $P'$ , it seems that the discovery is lucky to a fairly high degree.
  - There are many microconditions in which it rains, so William ultimately plants roses in a pot on his deck.
  - In others, Glory has a change of heart, and retrieves her treasure.
3. Again, the contextually-determined threshold is probably low enough for the discovery to be lucky simpliciter.

# SAL and BURIED TREASURE

Luckiness varies with nomological regularity.

- If pedological regularities raise the probability of discovery, then the discovery is correspondingly less lucky.
  - Similarly for psychological regularities of William's or Glory's desires.
- If there are no such regularities, then the discovery is correspondingly more lucky.

## SAL and RELIABLE NEWSPAPER

- According to the SAL, there is a fact of the matter about the degree to which  $E_1$  is lucky.
  - We might not be able to discern that degree, but there is a fact of the matter about it.
- Context then determines whether  $E_1$  is lucky simpliciter.

$E_1$ : the event of Emily forming the belief that  $b_1$ .

Introduction

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The PAL

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Counterexamples to the PAL

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The SAL

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Advantages of the SAL

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Thank you

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Contact information: isaac.wilhelm@rutgers.edu

## Phase Space

- Each point in the universe's phase space represents a physically possible way that the particles of the universe could be arranged.
  - Thus, each point is a physically possible world.
- Think of phase space as the set of all  $6n$ -tuples.
  - $n$  is the number of particles in the universe.
  - For each particle, six numbers completely specify its physical state: its position and its velocity in the  $x$ -direction,  $y$ -direction, and  $z$ -direction.

## Macrostates and Microstates

- Let  $M$  be the universe's macrostate, and let  $R_M$  be the corresponding region of phase space.
- One point in  $R_M$ —call it ' $a$ '—represents the exact microcondition of the universe.
  - The point  $a$  represents the exact distribution of particle positions and particle velocities.
- Other points in  $R_M$  represent microscopically distinct arrangements of particles that give rise to the *same* macrostate as  $a$  (namely, macrostate  $M$ ).
  - For example, one point in  $R_M$  represents the actual distribution of particles in the universe, except that one of the particles in my glass of water is moving a little slower than it actually is.
- All the points in phase space, but outside  $R_M$ , correspond to macroscopically distinguishable states of the universe.



## Objection 1

Objection: the notion of a macrostate is just as imprecise as the notion of a 'relevant' initial condition.

Reply: the notion of a macrostate is somewhat imprecise, because the carving is up to us. But there are robust sciences of those macrofacts, and our carving helps the special sciences succeed.

## Objection 2

Objection: luck cannot be identified with chance because luck can occur in deterministic worlds.

Reply: if probability is understood in the way described by the SAL, then chances—and therefore, luck—*can* occur in worlds with deterministic laws.

## Objection 3

Objection: luck is sensitive to modal closeness, not probability (Pritchard, 2014). The evidence for this includes

- intuitions about particular cases, and
- some studies from empirical psychology.

## Reply

The intuitive cases are not compelling.

- There may well be close possible worlds in which Emily's belief that  $b_1$  is false.

The empirical studies are consistent with probabilistic analyses of luck.

- Teigen's study (2005) can be explained by pointing to different events in the experiments that could have affected subjects' luck attributions.
- It can also be explained by some false inferences people tend to draw when reasoning about probabilities.