D&P: 11.3

3. For two proportions:
   a. Setup:
      i. See proportion $p_1$ with quality from pop. 1 in sample of size $n_1$, estimating $\pi_1$
      ii. See proportion $p_2$ with quality from pop. 2 in sample of size $n_2$, estimating $\pi_2$
   b. Test $H_0 : \pi_1 = \pi_2$
      i. SE $\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}$
         • Under $H_0$ don’t have two $p$’s, but only one:
         • estimate using common $p$ as total number with quality/total in both samples.
         • $= (n_1p_1 + n_2p_2)/(n_1 + n_2)$
   c. CI for $\pi_1 - \pi_2$?
      i. Estimate by $p_1 - p_2$
      ii. SD $\sqrt{\pi_1(1-\pi_1)/n_1 + \pi_2(1-\pi_2)/n_2}$
      iii. SE $\sqrt{\hat{p}(1-\hat{p})/n_1 + \hat{p}(1-\hat{p})/n_2}$
      iv. CI $p_1 - p_2 \pm z^*SE$
      v. Acceptance of hearing implants by deaf:
         \begin{tabular}{l|c|c}
            Deaf from Birth & 175 & 225 \\
            Reject & 75 & 25 \\
         \end{tabular}
         • The estimated proportion difference is $0.1 - 0.3 = -0.2$.
         • The standard error of the proportion difference is $\sqrt{0.3 \times (1-0.3)/250 + 0.1 \times (1-0.1)/250 = 0.035}$. 
         • The 95% confidence interval for $\pi_2 - \pi_1$ is
         $-0.2 \pm 1.96 \times 0.035 = -0.2 \pm 0.069 = (-0.268, -0.132)$.
         • The test standard error of the statistic is 0.036.
         • The test statistic is $| -0.2/0.036 | = 5.774$.
         • The $p$-value is 0.
   vi. Example: Faults under standard and modified process for batches of material
   d. Note: Only works if $p_1n_1, (1-p_1)n_1, p_2n_2, (1-p_2)n_2 > 5$ (CI) or $p(n_1 + n_2), (1-p)(n_1 + n_2) > 5$ (test)
   e. Summary of normal–theory tests and CI’s
      i. If
         • Standard conditions hold:
            ▶ parameter we want to estimate is mean of sampling distribution of statistic
            ▶ sampling distribution is approximately normal
            ▶ we have an estimate of its standard deviation, called standard error
         • Hold if parameter is a population mean and statistic is sample average
      ii. Then
         • Confidence Interval
            ▶ Center is statistic value
            ▶ plus or minus $z$ value times SE
         • Test
            ▶ Divide difference between statistic and $H_0$ parameter value by SE
            ▶ Compare to $z$ value
         • Rules for getting parts

\begin{itemize}
  \item If there is a $t$-refinement of $z$, use it.
  \item If statistic is the sum or difference of two independent components, variances add.
  \item If null hypothesis adds information for calculating SE, use it.
\end{itemize}