E. Power for approximately $\chi^2$ tests

1. $T = X^\top X = \sum_{k=0}^{K-1} X_j^2$, $X_j \sim N(\mu_j, 1)$, independent, $H_0: \mu_j = 0 \forall j$

2. MGF for addend $k$ is $M(\tau, \mu_k) = \exp(\mu_k^2 \tau / (1 - 2\tau))(1 - 2\tau)^{-1/2}$

3. MGF for $T$ is $M(\tau) = \prod_{k=0}^{K-1} \exp(\mu_k^2 \tau / (1 - 2\tau))(1 - 2\tau)^{-1/2} = \exp(\omega \tau / (1 - 2\tau))(1 - 2\tau)^{-K/2}$ for $\omega = \sum_{k=0}^{K-1} \mu_k^2$.

4. $\omega$ is called the noncentrality parameter

5. Often statistics are of the form $Y^\top Y$ for $Y = AX$, where $A$ satisfies $x^\top A^\top Ax = x^\top x$ for all $x$.


7. Hence $\eta^\top \eta = \mu^\top \mu = \omega$

8. Goodness of Fit:
   a. Null proportions $\pi_0^k$
   b. Alternate proportions $\pi_A^k$
   c. Total sample size $N$.
   d. Under $H_A$, $E\left[ (X_k - N\pi_0^k) / \sqrt{N\pi_0} \right] = 
      E\left[ (X_k - N\pi_A^k) / \sqrt{N\pi_A} \right] + \sqrt{N}(\pi_A^k / \sqrt{\pi_A} - \pi_0^k / \sqrt{\pi_0})$.
   e. So $\omega = N \sum_{k=0}^{K-1} (\pi_A^k / \sqrt{\pi_A} - \pi_0^k / \sqrt{\pi_0})^2 = $
\[ N \sum_{k=0}^{K-1} \left( \pi_k^A - \pi_k^0 \right)^2 / \pi_k^0. \]

f. Cohen calls \( \sqrt{\omega} \) before multiplying by \( N \) the effect size.

A: 7-7.1

VIII. Models and Graphs

A. The rule: Never put a term in the model without lower order terms

1. Ex. all models we examine effectively have an intercept

2. Ex. a model with \( X \times Y \) interactions must also have \( X \) and \( Y \) main effects

3. Ex. a model with \( X \times Y \times Z \) interactions must also have \( X \), \( Y \), \( Z \) main effects, \( X \times Y \), \( X \times Z \), \( Y \times Z \) interactions.

B. The Picture:

1. Make dots (vertices) corresponding to main effects in the model

2. Join dots by lines (edges) if associated two-way interaction is in the model.

   a. Edges are undirected.

3. Examples for model with four variables \( W, X, Y, Z \) are in Figure 11/.

C. Graphical models:

1. Sets of vertices connected all to each other are called cliques.
2. A model is called graphical if higher-order interactions are present to connect cliques, and are absent otherwise.

D. Conditional Independence:
1. A path is a sequence of edges leading from one vertex to another.

2. If all paths leading from one vertex to another vertex run through a third vertex, then the associated variables for the first two vertices are independent conditional on the third vertex.

3. Works for sets of variables rather than just variables if model is graphical.

4. Example: \[ W \rightarrow X \rightarrow Y \]

   a. Hence model is \[ \log(\lambda_{wxy}) = \alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^W + \alpha_{xy}^Y \]

   b. In full multinomial model, \( P[W = w, X = x, Y = y] = \frac{\exp(\alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^W + \alpha_{xy}^Y)}{C} \) for \( C = \sum_s, t, y \exp(\alpha_s^W + \alpha_t^X + \alpha_y^Y + \alpha_{st}^W + \alpha_{ty}^X) \).

   c. In full multinomial model, \( P[X = x] = \exp(\alpha_x^X)\sum_s \exp(\alpha_s^W + \alpha_{sx}^W)\sum_u \exp(\alpha_u^Y + \alpha_{yu}^Y)]/C. \)

   d. In full multinomial model, \( P[W = w, Y = y|X = x] = \frac{\exp(\alpha_w^W + \alpha_{wx}^W)}{\sum_s \exp(\alpha_s^W + \alpha_{sx}^W)} \cdot \frac{\exp(\alpha_y^Y + \alpha_{xy}^Y)}{\sum_t \exp(\alpha_t^Y + \alpha_{xt}^Y)}. \)

   e. \( W \perp Y|X \) (i.e., \( W \) is independent of \( Y \) conditional on \( X \) ).