E. Power for approximately χ² tests
1. \( T = X^T X = \sum_{j=1}^{K-1} X_j^2, \ X_j \sim N(\mu_j, 1) \), independent, \( H_0: \mu_j = 0 \forall j \)
2. MGF for addend \( k \) is \( M(\tau, \mu_k) = \exp(\mu_k^2 \tau / (1 - 2\tau)) (1 - 2\tau)^{-1/2} \)
3. MGF for \( T \) is \( M(\tau) = \prod_{k=0}^{K-1} \exp(\mu_k^2 \tau / (1 - 2\tau)) (1 - 2\tau)^{-1/2} \) for \( \omega = \sum_{k=0}^{K-1} \mu_k^2 \).
4. \( \omega \) is called the noncentrality parameter.
5. Often statistics are of the form \( Y^T Y \) for \( Y = AX \), where \( A \) satisfies \( x^T A^T Ax = x^T x \) for all \( x \).
6. Let \( \eta = E[Y] = A \mu \).
7. Hence \( \eta^T \eta = \mu^T \mu = \omega \).
8. Goodness of Fit:
   a. Null proportions \( \pi_k^0 \)
   b. Alternate proportions \( \pi_k^A \)
   c. Total sample size \( N \).
   d. Under \( H_A \), \( E \left[ (X_k - N \pi_k^0) / \sqrt{N \pi_k^0} \right] = \)
   \( E \left[ (X_k - N \pi_k^A) / \sqrt{N \pi_k^A} + \sqrt{N (\pi_k^A / \sqrt{\pi_k^A} - \pi_k^0 / \sqrt{\pi_k^0})^2} \right] = \) \( N \sum_{k=0}^{K-1} (\pi_k^A / \sqrt{\pi_k^A} - \pi_k^0 / \sqrt{\pi_k^0})^2 \)
   e. So \( \omega = N \sum_{k=0}^{K-1} (\pi_k^A / \sqrt{\pi_k^A} - \pi_k^0 / \sqrt{\pi_k^0})^2 \)
   f. Cohen calls \( \sqrt{\omega} \) before multiplying by \( N \) the effect size.
   A: 7-7.1

VIII. Models and Graphs

Lecture 8

**Fig. 11: Graphical Representation of Some Models**

All models contain main effects

Z

∅

W

Z

∅

W

No interactions

W, X, Y interactions

W

∅

X

Y

∅

W

X

Y

W

Z

X

Y

Z

W

X

Y

Z

W, X, Y, Z interactions

\( \alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^W + \alpha_{xy}^X \)

b. In full multinomial model, \( \Pr [W = w, X = x, Y = y] = \exp(a_w^W + a_x^X + a_y^Y + a_{wx}^W + a_{xy}^X) / C \) for \( C = \sum_{s,t,u} \exp(a_s^W + a_t^X + a_u^Y + a_{st}^W + a_{su}^X + a_{tu}^Y) / C \).

c. In full multinomial model, \( \Pr [X = x] = \exp(a_x^X) / \sum_s \exp(a_s^W + a_t^X + a_u^Y + a_{st}^W + a_{su}^X + a_{tu}^Y) / C \).

d. In full multinomial model, \( \Pr [W = w, Y = y | X = x] = \exp(a_w^W + a_{wx}^W) / \sum_t \exp(a_t^X + a_{su}^X) / C \)

\( \sum_t \exp(a_t^X + a_{su}^X) / \sum_t \exp(a_t^X + a_{su}^X) \)

e. \( W \perp Y | X \) (ie., \( W \) is independent of \( Y \) conditional on \( X \))

3. Works for sets of variables rather than just variables if model is graphical.

4. Example: \( W \) \( X \) \( Y \)
   a. Hence model is \( \log(\lambda_{wxy}) = \)