X. Measuring Agreement

A. Two-Way Setup:
1. $X_{jk}$ are independent $P(\lambda_{jk})$
2. We are interested in hypotheses like $\lambda_{j+} = \lambda_{+j} \forall j$
   a. hypothesis is implied by $\lambda_{jk} = \lambda_{kj} \forall i, k$
   b. Converse holds only if $J = 2$
   c. Model under null hypothesis has $\hat{\pi}_{jk} = (X_{jk} + X_{kj})/2$

B. We began investigation using McNemar’s test
1. Items categorized in table are pairs
2. Row represents category for one entry in pair
3. Column represents category for other entry in pair
4. Pair identifier is represented only through who is hooked to who
5. Treatment/control status is represented by which measure gets put on which dimension (row or column)

C. Use independent Poisson model: Let $\eta_{jk} = \log(E[X_{jk}])$
1. $\eta_{jk} = \lambda^X_{j} + \lambda^Y_{k} + \lambda^{XY}_{jk}$
2. Symmetry holds if $E[X_{jk}]$ are symmetric
3. Recall that model is overparameterized
   a. Fix by setting $\lambda^X_{j} = \lambda^X_{k} = 0$
   b. Fix by setting $\lambda^{XY}_{jk} = \lambda^{XY}_{kj} = 0$
   c. Fix by setting contrasts to zero.
      i. A contrast is a linear combination of model parameters that one either wants to estimate or to test whether they are zero.
4. Suppose that overparameterization is fixed symmetrically.

D. Test of symmetry:

E. Test of quasi-symmetry:
1. Allow potentially different main effects
   a. Use some technique to remove redundant interactions
   b. Let level 0 be baseline
   c. Intercept is $\eta_{00}$
   d. $\lambda^X_{j} = \eta_{j0} - \eta_{00}$
   e. $\lambda^Y_{k} = \eta_{0k} - \eta_{00}$
   f. $\lambda^{XY}_{jk} = \eta_{jk} - \eta_{j0} - \eta_{0k} + \eta_{00}$
2. Choose null hypothesis interactions $\lambda^{XY}_{jk} = 0$. This $H_0$ implies certain constraints:
   a. Implies $\eta_{ij} - \eta_{0j} - \eta_{i0} + \eta_{00} = \eta_{ji} - \eta_{0i} - \eta_{j0} + \eta_{00} \forall i, j > 0$
   b. Clearly holds for $i = 0$ or $j = 0$ as well.
   c. Implies $\eta_{ij} - \eta_{0j} - \eta_{i0} = \eta_{ji} - \eta_{0i} - \eta_{j0} \forall i, j$
   d. Implies $\eta_{ij} - \eta_{ji} = \eta_{0j} + \eta_{i0} - \eta_{0i} - \eta_{j0} \forall i, j$

3. Easy to check that constraints (2) imply $H_0$.
4. When testing $H_0$: quasisymmetry, vs. $H_A$: general alternative, $DF = (J - 1)(J - 2)/2$.
5. Do this depend on which group is baseline?
   a. Pick another baseline group $b$
   b. $\eta_{ij} - \eta_{0j} - \eta_{ib} - \eta_{ji} + \eta_{0i} = 0$
   c. Replace coefficients with both indices not known to be zero by (2).
   d. No.

6. No serious simplification for test.

F. Test of marginal symmetry
1. Condition is that $\lambda_{j+} = \lambda_{+j} \forall j$
   a. Summation is on raw scale rather than log scale
   b. This is not a typical contrast
   c. Use CATMOD to do this.

G. Test of quasi-independence:
1. Saturated model with only diagonal interactions non-zero.
2. Fit using GENMOD with contrasts.

H. Summary of tests in Figure 13.