XII. Latent Class Analysis

A. Setup

1. One unobserved categorical variable \( W \)
2. \( L \geq 2 \) observed categorical variables \( X_j \)
3. \( P[X = x|W = w] = \prod_{l=1}^{L} P[X_l = x_l|W = w] \)
4. Generally fewer parameters than saturated model.
5. Example: 4 manifest variables, each with two levels
   a. 2 class model has \( 2 \times 4 + 1 = 9 \) parameters.
   b. Corresponding saturated model has 15 parameters.
   c. 1 class model has 4 parameters
6. Model for \( I \times J \times \cdots \times L \) table with \( k \) classes can be
   made a submodel of the model with \( k - 1 \) classes by
   a. setting class probability to 0 or 1: 1 restriction.
   b. setting two sets of conditional probabilities equal:
      \( (I - 1) + (J - 1) + \ldots + (K - 1) \) restrictions
   c. Varying dimension of submodel makes it irregular, and
      standard \( \chi^2 \) likelihood ratio techniques don't work.