Port-of-Entry Inspection: Sensor Deployment Policy Optimization

Elsayed A. Elsayed, Christina M. Young, Minge Xie, Hao Zhang, and Yada Zhu

***Abstract*—This paper considers the problem of container inspection at a port-of-entry. Containers are inspected through a specific sequence to detect the presence of nuclear materials, biological and chemical agents, and other illegal shipments. The threshold levels of sensors at inspection stations of the port-of-entry affect the probabilities of incorrectly accepting or rejecting a container. In this paper, we present several optimization approaches for determining the optimum sensor threshold levels, while considering misclassification errors, total cost of inspection, and budget constraints. In contrast to previous work which determines threshold levels and sequence separately, we consider an integrated system and determine them simultaneously. Examples applying the approaches in different sensor arrangements are demonstrated.**

***Note to Practitioners*—Increased emphasis on container inspection at ports-of-entry prompted researchers to investigate methodologies that help practitioners in operating such inspection systems in order to reduce both the risk of accepting "undesired containers" as well as the cost of inspection. The threshold levels of the sensors can be adjusted by changing, for example, the power level of the X-ray machine or the count level of the sensors in order to make appropriate decisions. This paper provides methodologies for determining the optimum threshold levels of sensors and order of inspection that minimize the overall cost of the system. An approach for obtaining these values under budget and risk constraints is also included in this paper.**

***Index Terms*—Container inspection, gamma rays, large-scale systems, network reliability, probability, probability of false accept, probability of false reject, receiver operating characteristic curve, sensor threshold levels, sequences.**

I. Introduction

THE trade globalization and outsourcing of manufacturing goods have caused significant increases in the number of cargo containers being transported internationally. For example, each year more than 100 million cargo containers which constitute about 90% of the entire world's cargo crisscross international sea lanes and more than 95% of the non-North American foreign trade arrives into U.S. ports by ship. Slowing the flow long enough to inspect either all or a statistically significant random selection of imports would be economically intolerable [1]. Emphasis on improving the security of such containers prompted the development and installation of a wide range of inspection machines and sensors. These machines or sensors have different capabilities of detecting the presence of nuclear materials, biological agents, drugs, hazardous materials, and otherwise illegal shipments.

One of the most widely used techniques of noninvasively "seeing" into a container [2] is based on the use of electromagnetic (EM) waves such as radio waves, light, X-rays, γ-rays, etc. This technique is utilized in the VACIS (vehicle and cargo inspection system) device which combines two formerly separate scanning techniques. Gamma rays produce images, while radiation detection tracks radioactive signatures.

In another port-of-entry station, an X-ray system generates a high energy X-ray beam that traverses the container under inspection. A detector collects X-ray energy from the container and provides a signal indicative thereof. The signal is processed to detect the presence of very high X-ray attenuation within the container, which is indicative of nuclear weapons or materials [3]. Other equipment for detecting biological agents uses fluorescent particle counters for detecting airborne bacteria. In this case, the threshold used to make a detection decision is related to the count of these particles.

The limit applied to a measurement (raw number or obtained through signal processing) to make an accept/reject decision about a particular container attribute is represented as a threshold level. The threshold level is a set value in the domain of sensor measurements and has a direct impact on the classification of the container and the probability of making a "wrong" decision. Therefore, it is important to determine the optimum threshold level that minimizes the overall cost of inspection and making the "wrong" decision [4].

Following inspection a container is classified as acceptable (containing no suspicious material) or not. The consequence of such decision generates two types of errors (Type I and II) which are defined by the following hypotheses. The null hypothesis H_0 is that a given container is safe, and the alternate hypothesis H_1 is that it contains suspicious material. Type I error (also known as an "error of the first kind" or α error) is referred to as a "false reject" and corresponds to instances where a container is rejected or goes through extensive manual examination when, in fact, it has no suspicious contents. Type II error (also known as an "error of the second kind" or β error) is referred to as a "false accept" and corresponds to instances where a container that has suspicious contents is accepted.

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The Neyman–Pearson type criterion can be used to model systems with these two types of errors. Reference [5] considers a Neyman–Pearson formulation where one assumes a bound on the global probability of false alarm, the goal is to determine the optimum local and global decision rules that minimize the global probability of miss (or equivalently, maximize the global probability of detection). When the inspection stations are deployed so that their observations are independent, one can show that these decision rules are threshold rules based on likelihood ratios [5]. The problem now becomes one of determining the optimal thresholds of the sensors at each station. While this task is quite nontrivial, it can still be done for a reasonably small number of inspection stations using iterative techniques or using complete enumeration [6]–[9].

The problem of container inspection, without considering the threshold level of the sensors at inspection stations, is known as the *sequential diagnosis problem* in which the attributes of the container are inspected, one by one, in order to find out the status of the container at the current inspection station. An inspection strategy S is a rule that specifies on the basis of the states of the attributes already inspected which characteristics are to be inspected next or stops inspection by recognizing the correct status of the container. Such an inspection strategy can naturally be represented as a binary decision tree and is well documented in literature [10]. This function is simple when the container is inspected and its function is represented by a simple series or simple parallel inspection system. However, when the system is composed of a large number of inspection stations such as parallel-series, series-parallel, and *k-out-of-n* decision (presence of k attributes out of a total n attributes), the Boolean function becomes difficult and in some of these cases the problem is classified as NP-complete [11].

This work represents a unique approach to the formulation of the port-of-entry inspection problem as an analytical model. Moreover, in contrast to previous work which determines threshold levels and sequence separately, we consider an integrated system and determine them simultaneously.

The port-of-entry inspection problem is decomposed into two subproblems. One problem deals with the determination of the optimum sequence of inspection or the structure of the inspection decision tree in order to achieve the minimum expected inspection cost. This problem can be formulated and investigated using approaches parallel to those used in the optimal sequential inspection procedure for reliability systems as described in [12]–[19]. The other problem deals with the determination of the optimum thresholds of the inspection stations so as to minimize the cost associated with false reject and false accept. In this paper, we give an overview of the solution to the first (sequence) problem and apply those results in obtaining an overall inspection policy solution by determining the optimum sequence of inspection for a given set of threshold levels at inspection stations.

This paper is organized as follows. Section II describes the port-of-entry container inspection problem. Section III develops a procedure to find the optimal solution minimizing the cost of misclassifications and another procedure to minimize the cost of inspection, and then combines the two approaches to minimize total cost. This section also introduces the receiver operating characteristics (ROCs) curve to illustrate the inherent tradeoff in the optimization problem of container classification and this work is also extended to include optimization under budget constraints. Section IV presents numerical examples of the methods and finally the last section offers a discussion of the work presented.

II. Problem Description

A. Port-of-Entry Container Inspection System

The inspection of a container at a port-of-entry is performed sequentially at stations that form an inspection system. Containers are inspected and classified according to observations made regarding their attributes. Suppose there are n inspection stations in the system; one sensor (equipment) at each station is used to identify one attribute of the container being inspected, for example presence of radiation or uncharacteristic X-ray readings. One reading is taken and compared to a prespecified threshold level to make a pass or fail decision for that particular attribute.

There are several categories into which we seek to classify the containers. In the simplest case, these are negative and positive, 0 or 1, with "0" designating containers that are considered "acceptable" and "1" designating containers that raise suspicion and require special treatment. After each inspection, we either classify the container as acceptable or subject it to another inspection process.

The classification is thought of as a system decision function F that assigns to each binary string of decisions $(d_1, d_2, \ldots, d_n)$ a category. In this paper, we focus on the case where there are only two categories. In other words $F(d_1, d_2, \ldots, d_n) = 0$ indicates negative class and that there is no suspicion with the container and $F(d_1, d_2, \ldots, d_n) = 1$ indicates positive class and that additional inspection is required, usually manual inspection.

In this paper, we define, for instance, a system that uses a series Boolean function as applying a decision function F that assigns the container class "1" if any of the individual decisions are fail, $d_i = 1$ for any station i. For example, $F(d_1, d_2, \ldots, d_n) = (d_1 \vee d_2 \vee \ldots d_n)$. Furthermore, a system that uses a parallel Boolean function is defined as applying a decision function F that assigns the container the class "1" only if all of the individual decisions are fail. For example, $F(d_1, d_2, \ldots, d_n) = (d_1 \wedge d_2 \wedge \ldots d_n)$.

B. Sensor Performance and Inspection Threshold Levels

Consider a large group of containers as a population of interest, some of which are unacceptable and should be rejected. Let π represent the portion of unacceptable containers, $0 \leq \pi < 1$, thus $100 \times \pi$ percent of the containers should ideally be rejected. For a container randomly selected from this population, let x represent its true status, with $x = 1$ representing that it should be rejected and $x = 0$ representing that it should be accepted. We have $P(x = 1) = \pi$ and $P(x = 0) = 1 - \pi$.

Suppose this container is placed for inspection in an inspection system consists of k sensors or inspection stations. Let r_i be its measurement taken by the ith sensor, $i = 1, 2, \ldots, k$. Generally speaking, this measurement r_i can be either numerical (continuous or discrete) or even graphical, as described in

Section I. To simplify the presentation of our development, however, we assume that r_i is numerical and normally distributed. We choose to use the normal distribution because normally distributed data are the most commonly seen data in practice and it has been used in port-of-entry inspection applications [8], [20], [21]. Many other distributions can be transformed to be normally distributed or approximately normally distributed. We use the normal model to illustrate our development.

In addition to the normal assumption, we expect the distribution of the measurement r_i to vary depending on the true status of the container, either $x = 1$ or $x = 0$. To reflect this relationship, we assume, in particular, that $r_i \sim N(\mu_{0i}, \sigma_{0i}^2)$ when $x = 0$ and $r_i \sim N(\mu_{1i}, \sigma_{1i}^2)$ when $x = 1$, with $\mu_{0i} \neq \mu_{1i}$. Here, (μ_{0i}, μ_{1i}) and $(\sigma_{0i}, \sigma_{1i})$ are the mean and standard deviation parameters of the two normal distributions, respectively, and they are assumed to be known or can be estimated from past inspection history. In the port-of-entry inspection problem, the goal is to distinguish between the two groups of containers with $x = 0$ and $x = 1$. Thus, the relative distance between μ_{0i} and μ_{1i}, but not their locations or scales, plays a key role in separating good and bad containers. Indeed, the problem is location and scale invariant (analogous to the problem of telling two temperatures apart without being affected by whether we use Celsius or Fahrenheit metric). Without loss of generality, we can simplify our notations by assuming $\mu_{0i} = 0$ and $\mu_{1i} = 1$. Otherwise, we set $r_i^{(\text{new})} = (r_i - \mu_{0i})/(\mu_{1i} - \mu_{0i})$, $\sigma_{0i}^{(\text{new})} = \sigma_{0i}/(\mu_{1i} - \mu_{0i})$ and $\sigma_{1i}^{(\text{new})} = \sigma_{1i}/(\mu_{1i} - \mu_{0i})$, so we have $r_i^{(\text{new})} \sim N(0, \sigma_{0i}^2)$ when $x = 0$ and $r_i^{(\text{new})} \sim N(1, \sigma_{1i}^2)$ when $x = 1$.

We consider in this paper the threshold approach. In this approach, each measurement r_i is compared against a given threshold value T_i. The ith station rejects this item $(d_i = 1)$ if the reading r_i is higher than T_i and accepts it $(d_i = 0)$ if the reading is less than T_i. In this decision making process at the station level, there are two types of potential errors. For a randomly selected container with status $x = 0$, there is a chance that a decision $d_i = 1$ is made at the ith inspection. This is a Type I error of falsely rejecting a good item $x = 0$ and the probability of committing such an error can be computed by $P(d_i = 1|x = 0) = P(r_i > T_i|x = 0) = 1 - \Phi((T_i)/(\sigma_{0i}))$. Also, for a container with true status $x = 1$, there is a chance that a decision $d_i = 0$ is made. This is a Type II error of falsely accepting a bad item $x = 1$. The probability of committing such an error is $P(d_i = 0|x = 1) = P(r_i \leq T_i|x = 1) = \Phi((T_i - 1)/(\sigma_{1i}))$.

C. System Inspection Policy

We consider the effect of inspection system parameters on the costs associated with performing inspection and misclassification of containers. It becomes clear that the overall performance of the inspection system is determined by both the sequence in which inspection stations are visited and the threshold levels applied at those stations, which we denote collectively as the inspection policy. Therefore, the goal is to formulate the expected cost of inspection and classification errors (false positive and false negative) and use this information to generate a policy for the system's optimum operation. It is assumed that the decision function is predetermined, while the sequence of stations visited and the sensor thresholds are decision variables of the optimization. The optimization method is illustrated for inspection systems with decision functions of series, parallel, series-parallel, and parallel-series Boolean functions. In principle, the optimization framework in this paper can be applied in considering the problem without the assumption of a predetermined decision function.

III. Optimization Approaches

A. Minimization of Cost of Misclassification

At the system level, there are also two types of misclassification errors: falsely reject a container that should be cleared and falsely accept a container that should be rejected. The probability of false reject (PFR) is defined as the probability of the overall system rejecting a container conditional on the true status being acceptable. The probability of false accept (PFA) is defined as the probability of the system accepting a container conditional on the true status being unacceptable. The complementary probabilities of these two errors are true reject (PTR) and true accept (PTA). Denote by D the decision of the entire inspection system of sensors, where $D = 1$ means to reject, and $D = 0$ to accept. The four probabilities are listed as follows:

$$\begin{aligned}\text{PFR} &= P(D = 1 \,|\, x = 0)\\ \text{PTA} &= P(D = 0 \,|\, x = 0) = 1 - \text{PFR}\\ \text{PFA} &= P(D = 0 \,|\, x = 1)\end{aligned}$$

and

$$\text{PTR} = P(D = 1 \,|\, x = 1) = 1 - \text{PFA}.$$

The inspection decision of a system D depends on the inspection results of its sensors and the system Boolean function. Some examples [22] are given next.

Example 1. Series System: In a system using a series Boolean decision function, if any one station returns a fail decision, the overall container fails immediately. The PFR and PFA for the k-series system are given by

$$\begin{aligned}\text{PFR}_{\text{series}}^{[k]} &= 1 - \prod_{j=1}^{k} P(d_j = 0 \,|\, x = 0)\\ &= 1 - \prod_{j=1}^{k} \Phi\left(\frac{T_j}{\sigma_{0j}}\right)\end{aligned}$$

and

$$\begin{aligned}\text{PFA}_{\text{series}}^{[k]} &= \prod_{i=1}^{k} P(d_i = 0 \,|\, x = 1)\\ &= \prod_{i=1}^{k} \Phi\left(\frac{T_i - 1}{\sigma_{1i}}\right).\end{aligned}$$

Example 2. Parallel System: In a system using a parallel Boolean decision function, if any one station returns a pass decision, the overall container is passed immediately. Thus, a con-

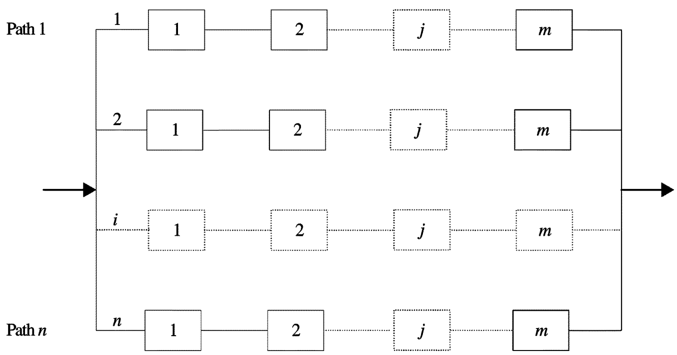

Fig. 1. Conceptual depiction of parallel-series system.

tainer must fail every station to fail overall. The PFR and PFA for the k-parallel system are given by

$$\mathrm{PFR}_{\mathrm{parallel}}^{[k]} = \prod_{j=1}^{k} P(d_j = 1 \,|\, x = 0) = \prod_{j=1}^{k} \left[1 - \Phi\left(\frac{T_j}{\sigma_{0j}}\right)\right]$$

and

$$\mathrm{PFA}_{\mathrm{parallel}}^{[k]} = 1 - \prod_{i=1}^{k} P(d_i = 1 \,|\, x = 1) = 1 - \prod_{i=1}^{k} \left[1 - \Phi\left(\frac{T_i - 1}{\sigma_{1i}}\right)\right].$$

Example 3. Parallel-Series System: In a system using a parallel-series Boolean decision function the container will fail overall only by failing at least one station in every parallel path. The PFR and PFA for the (n, m) parallel-series system (shown in Fig. 1) are given by

$$\mathrm{PFR}_{\mathrm{parallel-series}}^{[n,m]} = \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} P\left(d_{ij} = 0 | x = 0\right)\right] = \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} \Phi\left(\frac{T_{ij}}{\sigma_{0ij}}\right)\right]$$

$$\mathrm{PFA}_{\mathrm{parallel-series}}^{[n,m]} = 1 - \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} P(d_{ij} = 0 \,|\, x = 1)\right] = 1 - \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} \Phi\left(\frac{T_{ij} - 1}{\sigma_{1ij}}\right)\right].$$

Example 4. Series-Parallel System: In a system using a series-parallel Boolean decision function, the container will fail overall only by failing every station within at least one subsystem. The PFR and PFA for the (n, m) series-parallel system (shown in Fig. 2) are given by

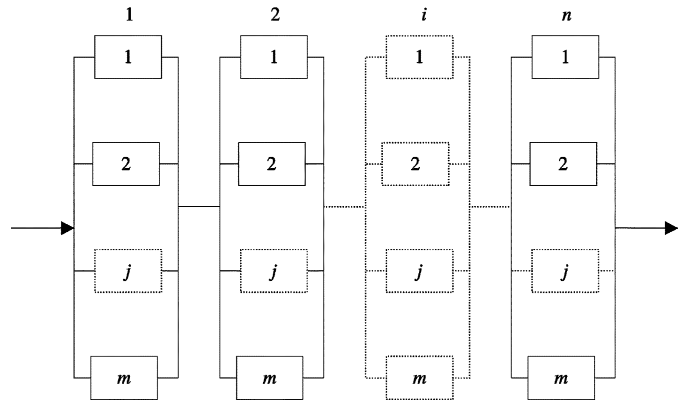

Fig. 2. Conceptual depiction of series-parallel system.

$$\mathrm{PFR}_{\mathrm{series-parallel}}^{[n,m]} = 1 - \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} P(d_{ij} = 1 \,|\, x = 0)\right] = 1 - \prod_{i=1}^{n} \left\{1 - \prod_{j=1}^{m} \left[1 - \Phi\left(\frac{T_{ij}}{\sigma_{0ij}}\right)\right]\right\}$$

and

$$\mathrm{PFA}_{\mathrm{series-parallel}}^{[n,m]} = \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m} P(d_{ij} = 1 \,|\, x = 1)\right] = \prod_{i=1}^{n} \left\{1 - \prod_{j=1}^{m} \left[1 - \Phi\left(\frac{T_{ij} - 1}{\sigma_{1ij}}\right)\right]\right\}.$$

It should be noted that for parallel-series and series-parallel systems in which the number of sensors per path is not constant (m varies by path), or the number of sensors per subsystem is not constant, the concepts of equations in Examples 3 and 4 would hold true, with slight changes in notation. In general, this extension is true wherever the parallel-series and series-parallel systems are discussed in this paper.

Ideally, we seek a set of threshold values for the sensors and a system configuration for which both these errors are minimized. Unfortunately, this simultaneous minimization of both PFR and PFA is not possible, and reducing one error is likely to increase the other. Therefore, we need to define an optimal policy that balances such a tradeoff.

One feasible approach is defining the expected cost of the system misclassifying a container. Let c_{FA} be the cost of the system accepting a "bad" container and c_{FR} be the cost of the system rejecting a "good" container. Then, the total cost of system classification error is

$$C_F = \pi \mathrm{PFA} c_{\mathrm{FA}} + (1 - \pi) \mathrm{PFR} c_{\mathrm{FR}}.$$

The set of optimal threshold values is the one that minimizes the expected cost C_F of the system misclassifying containers over all possible combinations of sensors in the system and all possible threshold values

$$\{T_1, T_2, \ldots, T_k\} = \arg\min C_F. \quad (1)$$

Currently, this optimization problem is solved by complete enumeration of the possible combinations of discretized threshold level values and computation of C_F. As the number of inspection stations increases, the computational requirements increase significantly, rendering the complete enumeration approach impractical in some cases. Alternative approaches and heuristics should be considered.

The cost of false rejection is the cost of additional tests. In the practice of port-of-entry inspection, these additional tests mean inspecting the contents manually. This is quite expensive since it might involve several workers for several hours, causing delays in completing the inspection and reduction in the inspection system throughput. Measuring the costs of false acceptance c_{FA} is even more challenging. Indeed, the cost of missing a container that contains illegal drugs is not comparable to the cost of missing a container that holds a "dirty bomb." One way is to assign a large cost value, say a few hundred—or even more—times the cost of a false reject.

An alternative and more flexible approach that avoids assigning exact values to the misclassification costs is the commonly used ROCs curve method. The ROC analysis was first developed in psychophysics to summarize data from signal detection experiments [23]. It has since been widely used in medicine, psychology, radiology, bio engineering, machine learning and data mining, among others. Depending on the practice, there are variations on how to produce an ROC curve, but all have the common feature of displaying two competing risks graphically as the parameter and condition changes. The most commonly seen ROC curves are in binary classifier systems, where the curve plots sensitivity values against (1—specificity) values. Here, sensitivity refers to the probability of classifying an item positive when it is indeed positive and specificity is the probability of classifying an item negative when it is indeed negative.

In the current context of port-of-entry inspection, an ROC curve plots the probability of true reject $(\mathrm{PTR} = 1 - \mathrm{PFA})$ against the probability of false reject (PFR), while varying the threshold parameters and the sequence of sensors. This kind of ROC curve provides a graphical representation of the trade-off between the probabilities of false accept (FA) and false reject (FR). It is a flexible and useful tool in decision-making [24].

Fig. 3 is an example of an ROC curve produced from a three sensor parallel system in the port-of-entry problem. Each point in the plots (a) or (b) represents a pair of (PFR, PTR) values for a given set of threshold level values and a specific sequence of the sensors. The most upper-left points form a curve which is referred to as the ROC curve. Theoretically speaking, in extreme cases, the ROC curve passes through two points (0, 0) and (1, 1). For any point not on the ROC curve, we can find a point on the ROC curve whose PFR or PFA or both values are better than those of the point that is not on the ROC curve. Thus, the ROC curve is the optimal curve in the sense of Pareto optimization.

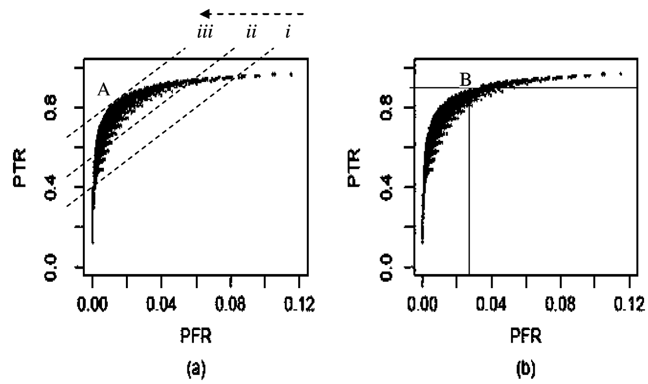

Fig. 3. ROC curve of a parallel system of three sensors.

The ROC curve consists of the best choices of threshold values under different preferences.

The ROC curve is closely related to the aforementioned optimization problem (1). In particular, for a given set of $(c_{\mathrm{FA}}, c_{\mathrm{FR}})$ values in the optimization problem, there is a point on the ROC curve that corresponds to it. The three broken parallel lines i, ii, and iii in Fig. 3(a) have a slope equal to $((1-\pi)/\pi) \times (c_{\mathrm{FR}})/(c_{\mathrm{FA}})$, and they can be used to depict the process of optimization in problem (1). As the parallel lines move from i to iii, the corresponding total cost of misclassification C_F decreases for those points on the lines. The solution to the optimal problem (1) corresponds to the point A on the ROC curve, at which the tangent line iii intercepts the ROC curve. The extreme point (0, 0) on the ROC curve corresponds to $c_{\mathrm{FA}} = 0$ and $c_{\mathrm{FR}} = \infty$, where the classifier finds no positives (detects no alarms). In this case, it always classifies the negative cases correct but it classifies all positive cases wrong. The extreme point (1, 1) on the other end corresponds to $c_{\mathrm{FA}} = \infty$ and $c_{\mathrm{FR}} = 0$, where all containers are classified as positive. So, all positive cases are correctly classified but all negative cases are misclassified (i.e., it raises a false alarm on each negative case).

The ROC curve can be utilized in the port-of-entry inspection problem to assist in decision making, typically by choosing an "operating" point (a fixed point) on the ROC curve. The goal is to find the best tradeoff between failing to detect positives against raising false alarms under given conditions. An illustrative example follows. Suppose we want to set a small tolerance level for the false acceptance rate (PFA) and, among those parameters that satisfy this constraint, choose a set that minimizes the false rejection rate (PFR). The tolerance constraint on PFA corresponds to the PTR constrained above the horizontal line as illustrated in Fig. 3(b) with tolerance level at 10%. Among those points above the horizontal line, the one that has the smallest PFR is at the intersection (point B) of the horizontal line and the ROC curve; see Fig. 3(b). This point B is the operating point of this problem that minimizes PFR at about 27% while holding the PFA smaller than the preset tolerance value. The set of threshold values and sensor sequence that correspond to this operating point is the solution for this constraint optimization problem. Note that this approach does not involve the selection of the cost values $(c_{\mathrm{FA}}, c_{\mathrm{FR}})$.

B. Optimal Sequence for Expected Inspection Cost

In addition to the cost of making false decisions, there is also the cost of inspection itself. There are several ways to calculate the cost of obtaining a sensor reading. One would be to break down the cost of obtaining a sensor reading into two components: unit variable cost and fixed cost. The unit variable cost is just the cost of using the sensor to inspect one container, and the fixed cost is the cost of the purchase and deployment of the sensor itself. In many cases, the primary cost is the unit variable cost since many inspections are very labor intensive. The fixed cost is usually a constant and often does not contribute to the optimization functions, so for simplicity we disregard the fixed cost. Thus, the inspection cost is basically the expected cost of making observations for a container. Note that depending on the system configuration, a container may or may not be inspected by all sensors. The arrangement of the sensors is closely related to the overall inspection cost.

Denote p_i by

$$\begin{aligned}p_i = P(d_i = 0) &= \sum_{j=0}^{1}[P(d_i = 0 \,|\, x = j)P(x = j)] \\ &= (1-\pi)\Phi\left(\frac{T_i}{\sigma_{0i}}\right) + \pi\,\Phi\left(\frac{T_i - 1}{\sigma_{1i}}\right)\end{aligned}$$

and $q_i = 1 - p_i$. They are functions of threshold values T_i. Let c_i be the inspection cost of sensor i. Reference [9] proves the following theorem.

Theorem 1:

a) For a series Boolean decision function, inspecting attributes $i = 1, 2, \ldots, n$ in sequential order is optimum in that it minimizes the expected inspection cost if and only if: $c_1/q_1 \le c_2/q_2 \le \cdots c_n/q_n$ (condition 1a).
In this case, the expected inspection cost is given by

$$C_I = c_1 + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1} p_j\right] c_i.$$

b) For a parallel Boolean decision function, inspecting attributes $i = 1, 2, \ldots, n$ in sequential order is optimum in that it minimizes the expected inspection cost if and only if: $c_1/p_1 \le c_2/p_2 \le \cdots c_n/p_n$ (condition 1b).
In this case, the expected inspection cost is given by

$$C_I = c_1 + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1} q_j\right] c_i.$$

We generalize these results to systems with arrangements of parallel-series and series-parallel sensors, given in Theorem 2.

Theorem 2:

a) Consider a parallel-series decision function with n paths of m sensors each (see Fig. 1). If an inspection system with attributes $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$ arranged in parallel-series is optimal, it satisfies the following conditions: the inspection sequence of the series of sensors within each path should be arranged in the order of $c_{i,1}/q_{i,1} \le c_{i,2}/q_{i,2} \le \cdots c_{i,m}/q_{i,m}$, and the inspection sequence of parallel paths should be arranged in the order of $C_1/P_1 \le C_2/P_2 \le \cdots \le C_n/P_n$ (condition 2a). Here, C_i and P_i are the (minimal) inspection cost and the probability of acceptance of the ith path

$$\begin{aligned}C_i &= c_{i1} + \sum_{j=2}^{m}\left[\prod_{k=1}^{j-1} p_{ik}\right] c_{ij} \\ &= c_{i1} + \sum_{j=2}^{m} c_{ij} \prod_{k=1}^{j-1}\left[(1-\pi)\Phi\left(\frac{T_{ik}}{\sigma_{0ik}}\right) \right. \\ &\quad \left. + \pi\,\Phi\left(\frac{T_{ik}-1}{\sigma_{1ik}}\right)\right]\end{aligned}$$

and $P_i = P(D_i = 0) = \prod_{j=1}^{m} p_{ij}$. In this case, the minimal inspection cost is

$$\begin{aligned}C_I &= C_1 + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1}(1 - P_j)\right] C_i \\ &= C_1 + \sum_{i=2}^{n} C_i \prod_{j=1}^{i-1}\left(1 - \prod_{k=1}^{m} p_{jk}\right).\end{aligned}$$

b) Consider a series-parallel decision function that has n subsystems in series with m units in parallel in each subsystem (see Fig. 2). If an inspection system with attributes $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$ arranged in series-parallel is optimal, it satisfies the following conditions: the inspection sequence of the series of each subsystem should be arranged in the order of $c_{i,1}/p_{i,1} \le c_{i,2}/p_{i,2} \le \cdots c_{i,m}/p_{i,m}$, and the inspection sequence of parallel paths should be arranged in the order of $C_1/Q_1 \le C_2/Q_2 \le \cdots \le C_n/Q_n$ (condition 2b). Here, C_i and Q_i are the (minimal) inspection cost and the probability of rejection of the ith subsystem

$$\begin{aligned}C_i &= c_{i1} + \sum_{j=2}^{m}\left[\prod_{k=1}^{j-1} q_{ik}\right] c_{ij} \\ &= c_{i1} + \sum_{j=2}^{m} c_{ij} \prod_{k=1}^{j-1}\left[(1-\pi)\left\{1 - \Phi\left(\frac{T_{ik}}{\sigma_{0ik}}\right)\right\} \right. \\ &\quad \left. + \pi\left\{1 - \Phi\left(\frac{T_{ik}-1}{\sigma_{1ik}}\right)\right\}\right]\end{aligned}$$

and $Q_i = P(D_i = 1) = \prod_{j=1}^{m}(1 - p_{ij})$. In this case, the minimal inspection cost is

$$\begin{aligned}C_I &= C_1 + \sum_{i=2}^{n}\left(\prod_{j=1}^{i-1} P_j\right) C_i \\ &= C_1 + \sum_{i=2}^{n} C_i \prod_{j=1}^{i-1}\left\{1 - \prod_{k=1}^{m}(1 - p_{jk})\right\}.\end{aligned}$$

The optimal arrangement of sensors depends on the values of p's and q's, which are functions of the threshold values. Therefore, given a set of threshold values the optimum sequence is the one that satisfies the constraints 1a, 1b, 2a, or 2b stated in the preceding theorems.

C. Minimization of Expected Total Cost

In some situations, it is conceivable that we may consider the combined cost of inspection and container misclassification. The total expected cost $C_{\text{Total}} = C_I + C_F$ is calculated from the results of the previous sections. To facilitate the computation for different systems with a large number of sensors, we provide induction methods which calculate the cost (both C_I and C_F) in the Appendix.

The optimization problem now becomes finding a set of threshold values $\{T_1, T_2, \ldots, T_k\}$ that minimizes the total expected cost

$$\{T_1, T_2, \ldots, T_k\} = \arg\min C_{\text{Total}} \tag{2}$$

among the sets of threshold values that satisfy the constraints in Section III-B.

The optimal set of threshold values from this optimization may be different from those obtained from optimization by minimizing cost of misclassification errors or minimizing inspection cost. In the case of port-of-entry inspection, the optimal solution of the cost-combined optimization may be very close to the solution obtained by the former if C_I is much smaller than the cost of the system misclassifying a container. Note that, in practice, in the port-of-entry inspection, the cost of false positives is often the cost of additional testing, such as opening the container and manually inspecting its contents. This is quite expensive since it might involve several workers for hours, delays in completing the inspection, and reduction in the inspection system throughput as stated earlier in this paper. In comparison to routine inspection cost of unit testing such as neutron or gamma emissions detection, this FR cost is relatively high. The FA cost would be even greater, including a huge potential social or economic impact.

D. Optimization With Budget Consideration

At a given port-of-entry inspection station, the inspection practice is often constrained by budget. It is not possible to open and manually inspect every container or every cargo, which is by far the most accurate but an extremely costly inspection method. If the budget allows, we may want to allow more containers to be manually inspected which, in turn, affects the sensor inspection process. For example, if the budget is large, it is possible to set low threshold levels to increase the PTR of the sensor system, and flag more containers for further manual inspection.

In our formulation, the total budget in an inspection station covers both the initial cost of the inspection system and additional manual inspection cost. Therefore, the budget is defined by

$$\begin{aligned}\text{budget} &= C_I + C_{\text{manual}} \\ &= C_I + c_{\text{unpack}}[(1-\pi)\text{PFR} + \pi\text{PTR}]\end{aligned}$$

where C_I is the cost of initial system inspection, and c_{unpack} is the unit cost of additional manual inspection (unpacking the container).

Under the budget constraint, we maximize the probability of properly classifying suspicious containers passing through the entire inspection system, including the sensor inspection system and manual inspections. So, the optimization problem can be described as

$$\begin{aligned}&\{T_1, T_2, \ldots, T_k\} = \arg\max \text{PTR} \\ &\text{subject to}: \ \text{Budget} < B_0\end{aligned}$$

where B_0 is the maximum available budget for the inspection system, and $\{T_1, T_2, \ldots, T_k\}$ is selected from possible threshold level values. We can formulate the budget constraint optimization problem similarly for other considerations. For example, minimization of the cost of misclassification can be obtained by finding the argument of the minimum of C_F defined in Section III-A.

This optimization problem can be presented by a graphical technique similar to the ROC curve, especially when we want to investigate the impact of the budget constraint. For instance, it is informative to investigate the relation between the chance of missing a suspicious cargo or dirty bomb and the budget. So, we plot the $\text{PTR} = 1 - \text{PFA}$ against the total budget B_0, while varying the threshold values and the sequence of sensors in an inspection system; see Fig. 8 and Section IV for further details. The most upper-left points form a curve. This curve consists of points corresponding to optimal threshold values and best combination of sensors at different budget levels.

IV. System Analysis With Numerical Examples

This section includes examples of solutions to the combined optimization problem of total cost (inspection and misclassification). Whereas the ROC curve was introduced as a graphical illustration of the inherent tradeoff between PTR and PFR, the minimum cost problem (2) produces an exact solution. Numerical examples are provided for the Boolean functions parallel, series, parallel-series, and series-parallel. Graphs of the optimization results are also presented. In addition, a numerical example is presented for the PTR maximization problem with budget consideration discussed in Section III-D.

Fig. 4 presents the minimum total cost of an inspection policy given the following system information: parallel Boolean decision function, unit misclassification penalty costs $c_{\text{FR}} = 500$ and $c_{\text{FA}} = 100000$, unit inspection cost $c_1 = c_2 = c_3 = 1$, and distribution parameters $\mu_{0i} = 0, \mu_{1i} = 1, \sigma_{01} = 0.45, \sigma_{02} = 0.55, \sigma_{03} = 0.5$, and $\sigma_{1i} = 0.5$ $(i = 1, 2, 3)$. The results are arranged by varying T_1 values along the horizontal axis and each point represents an optimal combination of threshold values and sequence, with the total cost along the vertical axis. The data series are the result of varying the parameter π, the true portion of unacceptable containers.

Fig. 4 also illustrates that for a given set of parameters, there is an optimal sequence and threshold values that correspond to the minimum total cost. For example, given $\pi = 0.0002$, the combination of threshold values $T^* = \{T_1, T_2, T_3\} = [0.55, 0.45, 0.45]$ and the inspection sequence 3-1-2 results in the minimum total cost for a system implementing a parallel Boolean decision function. The variation of the π value can influence the optimal inspection sequence and the optimal threshold values.

Fig. 5 presents the results for a series Boolean decision function. The parameter values and presentation of results are the

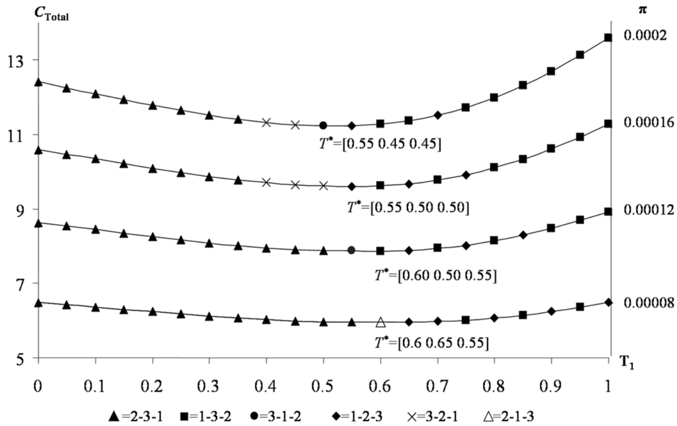

Fig. 4. Minimum cost curves for parallel system.

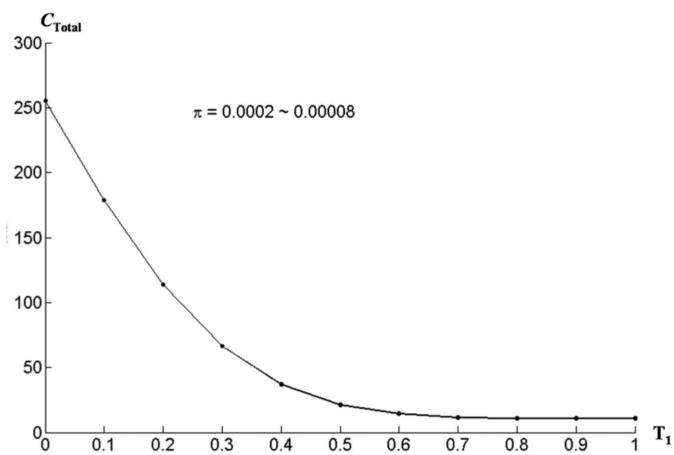

Fig. 5. Minimum cost curves for series system.

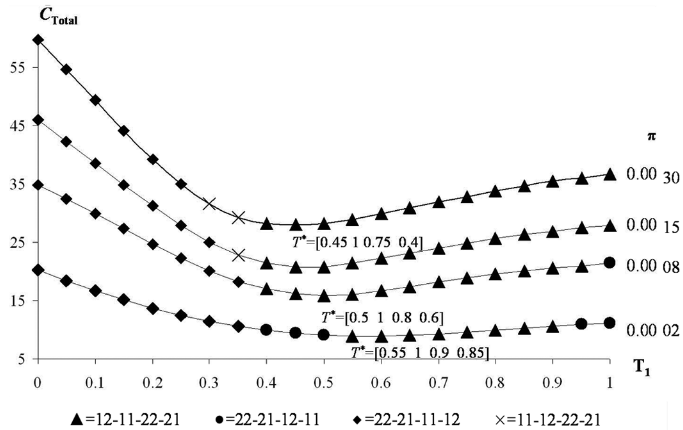

Fig. 6. Minimum cost curves for parallel-series system.

TABLE I
DISTRIBUTION PARAMETER VALUES FOR SENSORS IN PARALLEL-SERIES EXAMPLE

(Subsystem, Sensor)	σ_0	σ_1
(1,1)	0.25	0.35
(1,2)	0.65	0.25
(2,1)	0.45	0.55
(2,2)	0.55	0.35

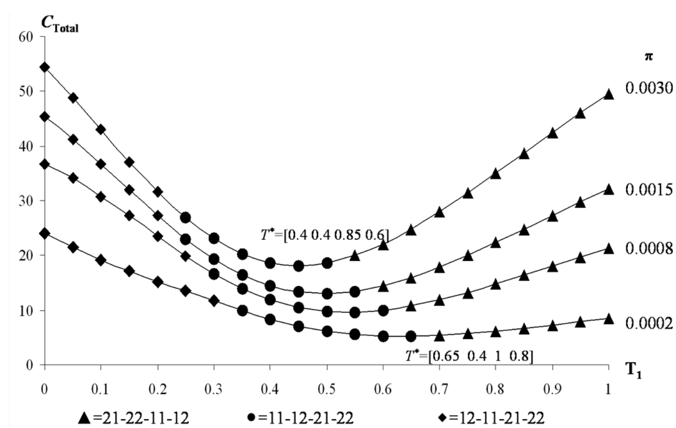

Fig. 7. Minimum cost curves for series-parallel system.

same as in the parallel Boolean example. In comparison to the results for the parallel Boolean decision function, the curves for various π values demonstrate significant overlap to the degree that the distinction between series is not apparent. This overlap indicates that the inspection sequence and threshold values for the series Boolean are not sensitive to the small π values in the example. In general, the total cost of the series system is higher than that of parallel for the given parameters. This is due to the relative increase in the expectation of a false rejection.

Fig. 6 presents the results for a parallel-series Boolean decision function of two subsystems in parallel, each consisting of two sensors in series. Table I presents the sigma values for each of the four sensors in this example. For all sensors, $\mu_0 = 0$ and $\mu_1 = 1$. The results are presented similarly to Figs. 4 and 5, with T_1 values across the horizontal axis and series resulting from various π values.

Fig. 7 presents the results for a series-parallel Boolean decision function. The sensor distribution parameters are the same as in the previous parallel-series example, presented in Table I.

With the same parameters in the series system, we plot the relationship between the budget level and the probability of true reject; see Fig. 8. If $B_0 = 1.35$ on the horizontal axis, the optimal threshold values are $\mathrm{T}^* = \{0.75\ 0.05\ 0.75\}$ and the probability of missing a suspicious cargo in the optimal case is about $1 - 0.9701 \cong 0.03$. If we increase the budget from $B_0 = 1.35$ to $B_0 = 1.50$, the optimal $\mathrm{T}^* = \{0.5\ 0.4\ 0.6\}$ and the probability of missing a suspicious cargo decreases to 0.0002. Clearly, the 11% increase of the budget results in a significant increase in the detection of unacceptable containers. Now, with $B_0 = 1.50$, an additional budget increase of the same amount results in little change in the probability of missing a suspicious cargo. It is not cost effective to apply the additional amount of inspection. Such information may help decision makers in assigning appropriate budgets to the port-of-entry inspection stations.

V. DISCUSSION

In this paper, we investigate the port-of-entry problem with a small number of inspection stations. Complete enumeration of all possible threshold levels for each sensor resulted in determining the optimum threshold levels for the sensors such that

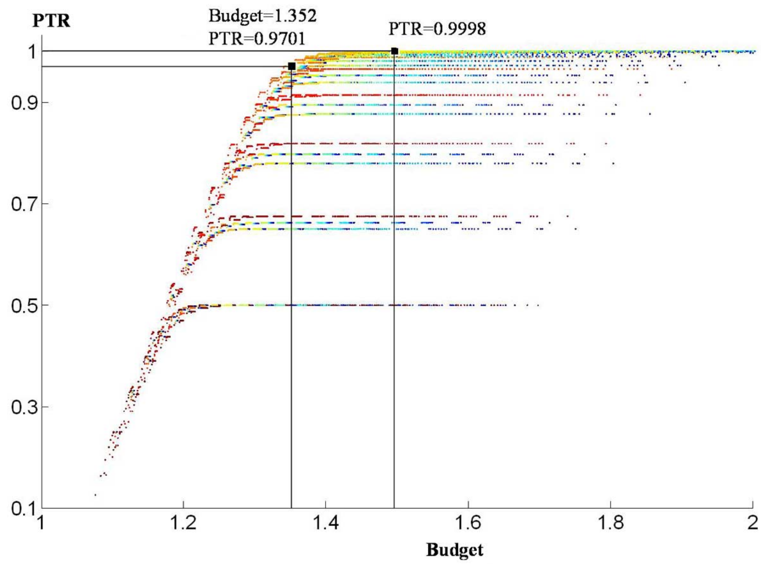

Fig. 8. Budget curve for parallel system.

the total cost is minimized. This has been done for sensors arranged in series, parallel, parallel-series, and series-parallel configurations. The key factor that has a direct effect on the determination of the sensors threshold levels is the cost of misclassification of a container with Type I error. This cost is difficult to estimate as it is a function of many unknowns but the effect could be catastrophic. Clearly, tightening the threshold levels minimizes the Type I error but may increase the cost of delaying the container. This has not been considered in this research but raises an important consideration of not only the cost of container misclassification but also the cost of delay incurred in the system.

Hence, we have two conflicting objectives, which lead to a multiobjective optimization problem, currently under investigation. Likewise, the optimum inspection sequencing problem in a multiobjective problem has not been addressed in this paper and it demands further work. The effect of measurement error on system performance and sensitivity analysis of system parameters present other areas for future research. Finally, the determination of the optimum threshold levels of sensors arranged in non standard arrangements such as a general network of sensors or k-out-of-n arrangement warrants further investigation.

Another point of interest is that in this paper we use a broad definition of containers considered unacceptable by border inspection authorities for various reasons. This is a basic assumption of the current model, assuming one π value to reflect all containers having any unacceptable contents. This model can be extended to consider individual types of risk with a different π_i value for each type. This extension would have significant effects which would be incorporated into the construction of the model. To include more individual assumptions of risks would require more historical data to generate information and distributions; however, more specific misclassification cost values associated with the various risks could be used. Future work involving π_i and cost values for separate risks is worth investigating.

APPENDIX

1) Induction Formula for the Parallel System: Suppose the $(k+1)$th sensor is added to the k-parallel sensors with $c_1/p_1 \leq \cdots \leq c_k/p_k \leq c_{k+1}/p_{k+1}$. For this system of $(k+1)$ parallel sensors, the cost of inspection can be calculated by the following induction formula:

$$C_I^{[1]} = c_1 \text{ and } C_I^{[k+1]} = C_I^{[k]} + c_{k+1} \prod_{j=1}^{k} q_j, \text{ for } k = 1, 2, \ldots$$

The cost function of making a false decision is

$$C_F^{[k+1]} = \pi c_{\text{FA}} \left(1 - A^{[k+1]}\right) + (1-\pi) c_{\text{FR}} B^{[k+1]}$$

where $A^{[k+1]}$ and $B^{[k+1]}$ can be computed by the following induction formulas:

$$A^{[1]} = \bar{\Phi}\left(\frac{T_1 - 1}{\sigma_{11}}\right), B^{[1]} = \bar{\Phi}\left(\frac{T_1}{\sigma_{01}}\right)$$

$$A^{[k+1]} = A^{[k]} \bar{\Phi}\left(\frac{T_{k+1} - 1}{\sigma_{1,k+1}}\right)$$

and

$$B^{[k+1]} = B^{[k]} \bar{\Phi}\left(\frac{T_{k+1}}{\sigma_{0,k+1}}\right), \text{for } k = 1, 2, \ldots \text{and } \bar{\Phi} = 1 - \Phi.$$

2) Induction Formula for the Series System: Suppose the $(k+1)$th sensor is added to the k-series sensors with $c_1/q_1 \leq \cdots \leq c_k/q_k \leq c_{k+1}/q_{k+1}$. For this system of $(k+1)$ series sensors, the cost of inspection can be calculated by the following induction formula:

$$C_I^{[1]} = c_1 \text{ and } C_I^{[k+1]} = C_I^{[k]} + c_{k+1} \prod_{j=1}^{k} p_j, \text{for } k = 1, 2, \ldots$$

The cost function of making a false decision is

$$C_F^{[k+1]} = \pi c_{\text{FA}} A^{[k+1]} + (1-\pi) c_{\text{FR}} (1 - B^{[k+1]})$$

where $A^{[k+1]}$ and $B^{[k+1]}$ can be computed by the following induction formulas:

$$A^{[1]} = \Phi\left(\frac{T_1 - 1}{\sigma_{11}}\right), B^{[1]} = \Phi\left(\frac{T_1}{\sigma_{01}}\right)$$

$$A^{[k+1]} = A^{[k]} \Phi\left(\frac{T_{k+1} - 1}{\sigma_{1,k+1}}\right)$$

and

$$B^{[k+1]} = B^{[k]} \Phi\left(\frac{T_{k+1}}{\sigma_{0,k+1}}\right), \text{for } k = 1, 2, \ldots$$

3) Induction Formula for the Parallel-Series System:

Step 1: Add $(m+1)$th sensor to each branch:

$$(n, m) \rightarrow (n, m+1).$$

If $c_{i,1}/q_{i,1} \leq \cdots \leq c_{i,m}/q_{i,m} \leq c_{i,m+1}/q_{i,m+1}$ for all branches, Section III-B gives the minimum inspection cost as

$$C_I^{[m+1]} = C_{I,1}^{[m+1]} + \sum_{i=2}^{m} C_{I,i}^{[m+1]} \prod_{j=1}^{i-1} \left[1 - \prod_{k=1}^{m+1} p_{j,k}\right]$$

where the inspection cost of the ith path of $m+1$ sensors in series $C_{I,i}^{[m+1]}$, for $i = 1, 2, \ldots, n$, can be calculated by the induction formula in Appendix A–2.

From the results from Section III-A, the total expected cost of misclassification is

$$C_F^{[m+1]} = \pi c_{\text{FA}} [1 - PM^{[m+1]}] + (1-\pi) c_{\text{FR}} PN^{[m+1]}$$

where $PM^{[m+1]} = \prod_{i=1}^{n} \{1 - M_i^{[m+1]}\}, PN^{[m+1]} = \prod_{i=1}^{n} \{1 - N_i^{[m+1]}\}$, and $M_i^{[m+1]}$ and $N_i^{[m+1]}$ can be updated by the following induction formulas:

$$M_i^{[m+1]} = M_i^{[m]} \Phi\left(\frac{T_{i,m+1} - 1}{\sigma_{1,(i,m+1)}}\right) \text{ and } N_i^{[m+1]} = N_i^{[m]} \Phi\left(\frac{T_{i,m+1}}{\sigma_{0,(i,m+1)}}\right).$$

Step 2: Add the $(n+1)$th branch with m sensors

$$(n, m) \rightarrow (n+1, m).$$

If $C_{I,1}/Q_1 \leq \cdots \leq C_{I,n}/Q_n \leq C_{I,n+1}/Q_{n+1}$, Section III-B gives the minimum inspection cost as

$$C_I^{[n+1]} = C_I^{[n]} + C_{I,n+1} G_n$$

where G_n can be updated by induction

$$G_1 = Q_1, \text{ and } G_{j+1} = G_j Q_j,$$

$$\text{with } Q_j = 1 - \prod_{s=1}^{m} \left[(1-\pi)\Phi\left(\frac{T_{js}}{\sigma_{o,js}}\right) + \pi\Phi\left(\frac{T_{js} - 1}{\sigma_{1,js}}\right)\right].$$

From the result of Section III-A, the total expected cost of misclassification is

$$C_F^{[n+1]} = \pi c_{\text{FA}} \left[1 - PA^{[n]} A_{n+1}\right] + (1-\pi) c_{\text{FR}} PB^{[n]} B_{n+1}$$

where $PA^{[n+1]}$ and $PB^{[n+1]}$ can be updated by induction formula, for $j = 1, 2, \ldots, N$

$$PA^{[1]} = A_1, PA^{[j+1]} = PA^{[j]} A_j,$$

$$PB^{[1]} = B_1, PB^{[j+1]} = PB^{[j]} B_j$$

and, for $j = 1, 2, \ldots, n+1$

$$A_i = 1 - \prod_{j=1}^{m} \Phi\left(\frac{T_{ij} - 1}{\sigma_{1,ij}}\right)$$

and

$$B_i = 1 - \prod_{j=1}^{m} \Phi\left(\frac{T_{ij}}{\sigma_{o,ij}}\right).$$

4) Induction Formula for the Series-Parallel System:

Step 1: Add the $(m+1)$th sensor in each subsystem:

$$(n, m) \rightarrow (n, m+1).$$

If $c_{i,1}/p_{i,1} \leq \cdots c_{i,m}/p_{i,m} \leq c_{i,m+1}/p_{i,m+1}$ for all the branches, Section III-B gives the minimum inspection cost as

$$C_I^{[n,m+1]} = C_{I,1}^{[m+1]} + \sum_{i=2}^{n} C_{I,i}^{[m+1]} \prod_{j=1}^{i-1} \left[1 - \prod_{s=1}^{m+1} (1 - p_{js})\right]$$

where the inspection cost of the ith path of $m+1$ sensors in series $C_{I,i}^{[m+1]}$, for $i = 1, 2, \ldots, n$, can be calculated by the induction formula in Appendix A–1.

From the result of Section III-A, the total expected cost of misclassification is

$$C_F^{[m+1]} = \pi c_{\text{FA}} \left(PA^{[m+1]}\right) + (1-\pi) c_{\text{FR}} \left(1 - PB^{[m+1]}\right)$$

where $PA^{[m+1]} = \prod_{i=1}^{n} \{1 - A_i^{[m+1]}\}, PB^{[m+1]} = \prod_{i=1}^{n} \{1 - B_i^{[m+1]}\}$, and $A_i^{[m+1]}$ and $B_i^{[m+1]}$ can be updated by induction formula

$$A_i^{[m+1]} = A_i^{[m]} \bar{\Phi}\left(\frac{T_{i,M+1} - 1}{\sigma_{1,(i,M+1)}}\right)$$

$$B_i^{[m+1]} = B_i^{[m]} \bar{\Phi}\left(\frac{T_{i,m+1}}{\sigma_{0,(i,m+1)}}\right), \text{with } \Phi = 1 - \bar{\Phi}.$$

Step 2: Add the $(n+1)$th subsystem with m sensors

$$(n, m) \rightarrow (n+1, m).$$

If $C_{I,1}/Q_1 \leq \cdots \leq C_{I,n}/Q_n \leq C_{I,n+1}/Q_{n+1}$, Section III-B gives the minimum inspection cost as

$$C_I^{[n+1]} = C_I^{[n]} + C_{I,n+1} G_n$$

where G_n can be updated by induction

$$G_1 = P_1, \text{and } G_{j+1} = G_j P_j, \\ \text{with } P_j = 1 - \prod_{s=1}^{m} \left[(1-\pi)\Phi\left(\frac{T_{js}}{\sigma_{o,js}}\right) + \pi\Phi\left(\frac{T_{js}-1}{\sigma_{1,js}}\right)\right].$$

From the result of Section III-A, the total expected cost of misclassification is

$$C_F^{[n+1]} = \pi c_{\text{FA}} PA^{[n]} A_{n+1} + (1-\pi) c_{\text{FR}} PB^{[n]} B_{n+1}$$

where $PA^{[n+1]}$ and $PB^{[n+1]}$ can be updated by induction formula, for $j = 1, 2, \ldots, n$

$$PA^{[1]} = A_1, PA^{[j+1]} = PA^{[j]} A_j, \\ PB^{[1]} = B_1, PB^{[j+1]} = PB^{[j]} B_j,$$

and, for $j = 1, 2, \ldots, n+1$

$$A_i = 1 - \prod_{j=1}^{m} \bar{\Phi}\left(\frac{T_{ij}-1}{\sigma_{1,ij}}\right) \\ B_i = 1 - \prod_{j=1}^{m} \bar{\Phi}\left(\frac{T_{ij}}{\sigma_{o,ij}}\right), \text{with } \Phi = 1 - \bar{\Phi}.$$

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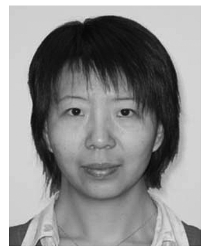

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