Note: The problems are taken from the Exercises in Casella and Berger (2002) unless otherwise noted. For each problem, please explain your reasoning clearly. It is not acceptable to only provide your final result.

**Homework 1** (Due Wed, Feb 1):
6.9, 6.10, 6.11, 6.12, 6.15, 6.20, 6.23, 6.30, 6.31(b,c)

The question 6.31(b)(ii) should be corrected as follows. Suppose $X_1, \ldots, X_N$ are $N$ Monte Carlo samples, where each $X^j$ consists of an iid sample of size $n$ from $N(\mu, \sigma^2)$. Let $M^j$ be the sample median and $\bar{X}^j$ the sample mean, both computed from $X^j$. Let $\bar{M}$ be the average of $(M^1, \ldots, M^N)$, and $\bar{X}$ be the average of $(\bar{X}^1, \ldots, \bar{X}^N)$. A naive estimator of the variance of the sample median is then

$$v_1 = \frac{1}{N - 1} \sum_{j=1}^{N} (M^j - \bar{M})^2.$$ 

The swindle estimator of the variance of the sample median is

$$v_2 = \frac{\sigma^2}{n} + \frac{1}{N - 1} \sum_{j=1}^{N} \{M^j - \bar{X}^j - (\bar{M} - \bar{X})\}^2.$$ 

Show that the variance of $v_1$ is approximately $2[\text{var}(M)]^2/(N - 1)$, and the variance of $v_2$ is approximately $2[\text{var}(M - \bar{X})]^2/(N - 1)$.

**Homework 2** (Due Wed, Feb 22):
6.37, 7.2, 7.6, 7.10, 7.12, 7.13, 7.18, 7.23, 7.24,
7.37, 7.38, 7.42, 7.44, 7.45, 7.46, 7.49, 7.52

Additional problem I:
Let $X_1, \ldots, X_n$ be an iid sample from $N(\mu, \sigma^2)$ with unknown $(\mu, \sigma^2)$.
1) Find the canonical parameters for $N(\mu, \sigma^2)$ as an exponential family.
2) Derive the maximum likelihood estimators of $(\mu, \sigma^2)$ by solving the score equation in terms of the canonical parameters.

Additional problem II:
Let $X$ be an observation from $N(\mu, 1)$ with unknown $\mu$. Suppose that the prior on $\mu$ is $N(0, \tau^2)$ with a fixed $\tau > 0$.
1) Show that the maximum likelihood estimator of $\mu$ is $\hat{\mu} = X$.
2) Find the posterior mean of $\mu$, denoted by $\bar{\mu}$, as a point estimator.
3) Find the bias, variance, and mean squared error of $\bar{\mu}$. [Continue to next page]
4) Determine the condition, depending on $\mu$ and $\tau$, such that the mean square error of $\tilde{\mu}$ is no greater than that of $\hat{\mu}$.

Additional problem III:
Let $(X_1, \ldots, X_n)$ be an iid sample from Uniform $(0, \theta)$ with unknown $\theta > 0$. Consider the estimator $\hat{\theta} = \frac{n+1}{n} X_{(n)}$.
1) Show that $\hat{\theta}$ is unbiased for $\theta$.
2) Show that
   \[
   \text{var}_\theta(\hat{\theta}) = \frac{\theta^2}{n(n + 2)}.
   \]

Homework 3 (Due Wed, March 29):
7.62, 7.65, 8.5, 8.6, 8.7(a), 8.11, 8.12, 8.15, 8.22, 8.23,
8.28, 8.29, 8.31, 8.40, 8.41, 8.47, 8.52, 8.53

Additional problem I:
Let $(X_1, \ldots, X_n)$ be an iid sample from $N(\mu_x, \sigma_x^2)$ and, independently, $(Y_1, \ldots, Y_m)$ be an iid sample from $N(\mu_y, \sigma_y^2)$, where $\sigma_x^2$ and $\sigma_y^2$ are known. Consider testing
   \[
   H_0 : \mu_x = \mu_y = 0,
   \]
   \[
   H_1 : \mu_x \neq 0 \text{ or } \mu_y \neq 0.
   \]
1) Find a likelihood ratio test (LRT) rejection region and a union-intersection test (UIT) rejection region to achieve size $\alpha$.
   For the remaining questions, set $\alpha = 10\%$.
2) Compute and plot the power function for LRT and UIT rejection regions along the direction $\mu_x = 0$ and $\mu_y \in \mathbb{R}$. You may use R or other language.
3) Compute and plot the power function for LRT and UIT rejection regions along the direction $\mu_x = \mu_y \in \mathbb{R}$.
4) Compute and plot the power function for LRT and UIT rejection regions for $(\mu_x, \mu_y) \in \mathbb{R}^2$. Choose a plotting method that you think most appropriate.

Homework 4 (Due Wed, Apr 12):

Homework 5 (Due Wed, May 3):
10.3, 10.9, 10.10(ab), 10.12, 10.29, 10.31, 10.35(b), 10.37(b),
10.17(d) [Verify the normality result, i.e, complete “tedious matrix calculations.”]