Assignment #2

6.26 It is given that $f(i) = \frac{1}{2}$, for $9 \le i \le 11$. If $P = 2i^2$, then $i = \sqrt{\frac{P}{2}}$ and

$$\left|\frac{di}{dp}\right| = \left(\frac{1}{4}\right)\left(\frac{p}{2}\right)^{-1/2} = \left(\frac{1}{2}\right)^{3/2} p^{-1/2}.$$

Then

$$f_P(p) = \frac{1}{2^{5/2}p^{1/2}} = \frac{1}{4\sqrt{2p}}$$

for $9 \le \sqrt{\frac{p}{2}} \le 11$ or in other words $162 \le p \le 242$.

6.34 Since Y_1 and Y_2 are independent standard normal random variables, the moment-generating functions for Y_1^2 and Y_2^2 can be written, from Example 6.11, as

$$m_{Y_1^2}(t) = \frac{1}{(1-2t)^{1/2}}$$
$$m_{Y_2^2}(t) = \frac{1}{(1-2t)^{1/2}}.$$

Now using Theorem 6.2, we have

$$m_U(t) = m_{Y_1^2}(t)\, m_{Y_2^2}(t) = \frac{1}{(1-2t)^1},$$

which is the moment-generating function of a gamma random variable with $\alpha = 1$ and $\beta = 2$. Hence by Theorem 6.1, U has a gamma distribution with $\alpha = 1$ and $\beta = 2$. Equivalently, U has a χ^2 distribution with 2 degrees of freedom.

6.38 By Theorem 6.3 we have $U = \sum_1^n Y_i \sim N(n\mu, n\sigma^2)$. Note that U is the total weight of watermelons in the packing container. Then

$$0.05 = P(U > 140) = P(Z > \tfrac{140 - n\mu}{n\sigma^2}).$$

Looking at the standard normal table shows that

$$\frac{140 - n\mu}{n\sigma^2} = 1.645$$

Thus, we can solve for n to obtain

$$n = \frac{140}{\mu + 1.645\sigma^2}.$$

Now, if we use $\mu = 15$ and $\sigma^2 = 4$ we see that $n = 6.5$. Therefore, the maximum number of melons that should be put in the container is 6.

6.68 **a.** This is similar to Exercise 6.67. Calculate

$$1 - F(y) = \int_y^\infty e^{-(t-\theta)}\, dt = e^{-(y-\theta)}$$

Then

$$g_1(y) = n\left[e^{-(y-\theta)}\right]^{n-1} e^{-(y-\theta)} = ne^{-n(y-\theta)}$$

for $y \ge 0$.

b. $E\left(Y_{(1)}\right) = \int_0^\infty nye^{-n(y-\theta)}\, dy = n\int_0^\infty (z+\theta)e^{-nz}\, dz$ (with $z = y - \theta$)

$$= n\Gamma(2)\left(\tfrac{1}{n}\right)^2 + n\theta\left(\tfrac{1}{n}\right)^1 = \tfrac{1}{n} + \theta$$

Notice that the integral is calculated in two parts, using the fact that the constants associated with the integrals must be constants of gamma random variables.