HOMEWORK #5

8.13 Note that

$$E(\hat{p}_1) = E\left(\frac{Y}{n}\right) = \left(\frac{1}{n}\right)(np) = p$$
$$E(\hat{p}_2) = E\left(\frac{Y+1}{n+2}\right) = \frac{1}{(n+2)}(np+1) = \frac{np+1}{n+2}$$

a. Bias $= \frac{np+1}{n+2} - p = \frac{np+1-np-2p}{n+2} = \frac{1-2p}{n+2}$.

b. $\text{MSE}(\hat{p}_1) = V(\hat{p}_1) + B^2 = V\left(\frac{Y}{n}\right) + 0 = \left(\frac{1}{n^2}\right) np(1-p) = \frac{p(1-p)}{n}$.

$$\text{MSE}(\hat{p}_2) = V(\hat{p}_2) + B^2 = V\left(\frac{Y+1}{n+2}\right) + \left(\frac{1-2p}{n+2}\right)^2$$
$$= \left[\frac{1}{(n+2)^2}\right] V(Y+1) + \frac{(1-2p)^2}{(n+2)^2}$$
$$= \frac{np(1-p)+(1-2p)^2}{(n+2)^2}.$$

c. We need to consider $\text{MSE}(\hat{p}_2) < \text{MSE}(\hat{p}_1)$.

$$\frac{np(1-p)+(1-2p)^2}{(n+2)^2} < \frac{p(1-p)}{n}$$
$$n^2p(1-p) + n(1-2p)^2 - p(1-p)(n+2)^2 < 0$$

This simplifies to

$$(8n+4)p^2 - (8n+4)p + n < 0$$

By the quadratic formula

$$p = \frac{8n+4 \pm \sqrt{(8n+4)^2 - 4(8n+4)n}}{2(8n+4)} = \frac{1}{2} \pm \sqrt{\frac{n+1}{8n+4}}$$

That is, p will be close to $\frac{1}{2}$.

8.42 a. $\hat{p} = \frac{268}{500} = .536$. Therefore, an approximate 98% confidence interval for p is

$$\hat{p} \pm z_{.01}\sqrt{\frac{\hat{p}\hat{q}}{n}} = .536 \pm 2.33\sqrt{\frac{(.536)(.464)}{500}} = .536 \pm .052 \text{ or } (.484, .588).$$

b. Since the interval does include $p = .51$, we cannot conclude that there is a difference in the graduation rates before and after Proposition 48.

8.60 a. We assume $p_1 = p_2 = .75$; $n_1 = n_2 = 1500$.

$$B = 2\sqrt{\frac{2(.75)(.25)}{n}} = .0316.$$

b. For a 90% confidence level, $z_{\alpha/2} = z_{.05} = 1.645$. Assuming equal sample sizes, we consider

$$1.645\sqrt{\frac{2(.75)(.25)}{1500}} = .02$$
$$\sqrt{n} = \frac{1.645\sqrt{2(.75)(.25)}}{.02}$$
$$n = 2536.89 \cong 2537.$$