8.100 The 99% confidence interval for μ is approximately

$$\overline{y} \pm t_{.005}\left(\frac{s}{\sqrt{n}}\right) = 79.47 \pm 2.947\left(\frac{25.25}{\sqrt{16}}\right) = 79.47 \pm 18.60$$

or $60.87 < \mu < 98.07$. Intervals constructed in this manner enclose μ 99% of the time in repeated sampling. Hence, we are fairly certain that this particular interval encloses μ.

8.104 Assume the scores are normally distributed with $\sigma_1^2 = \sigma_2^2$. Calculate

$$S_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{10(52)+13(71)}{11+14-2} = \frac{1443}{23} = 62.74$$

Then a 95% confidence interval for $\mu_1 - \mu_2$ will be

$$(\overline{y}_1 - \overline{y}_2) \pm t_{.025,23}\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = (64-69) \pm 2.069\sqrt{62.74\left(\frac{1}{11}+\frac{1}{14}\right)} = -5 \pm 6.60$$

or $(-11.60\ ,\ 1.60)$.

8.112 From Exercise 8.1,

$$\text{MSE}\left(S^2\right) = V\left(S^2\right) + \left[E\left(S^2\right) - \sigma^2\right]^2 = V\left(S^2\right) + 0 = \frac{2\sigma^4}{n-1}$$

Similarly,

$$\text{MSE}\left(S'^2\right) = V\left(S'^2\right) + \left[E\left(S'^2\right) - \sigma^2\right]^2 = \frac{(n-1)2\sigma^4}{n^2} + \left(\frac{n-1}{n} \times \sigma^2 - \sigma^2\right)^2$$

$$= \frac{2(n-1)\sigma^4}{n^2} + \frac{\sigma^4}{n^2} = \frac{(2n-1)\sigma^4}{n^2}$$

Look at

$$\text{MSE}\left(S'^2\right) - \text{MSE}\left(S^2\right) = \frac{(2n-1)\sigma^4}{n^2} - \frac{2\sigma^4}{n-1} = \frac{\sigma^4(1-3n)}{(n-1)n^2}$$

For $n > 1$, the quantity $(1-3n)$ will be negative, while σ^4, $n-1$, and n will be positive. Hence

$$\text{MSE}\left(S'^2\right) - \text{MSE}\left(S^2\right) < 0$$

or

$$\text{MSE}\left(S'^2\right) < \text{MSE}\left(S^2\right).$$

Which indicates that S'^2 may be a better estimator.