HOMEWORK #9

**9.90** In Exercise 9.73, $\overline{Y}^2$ was shown to be the MLE of $\theta^2$. Note, $t(\theta) = \theta^2$ and $\frac{\partial t(\theta)}{\partial \theta} = 2\theta$. Further we have $f(y|\theta) = \frac{1}{\theta} e^{-y/\theta}$, implying $\ln[f(y|\theta)] = -\ln\theta - \frac{y}{\theta}$. Then

$$\frac{\partial \ln[f(y|\theta)]}{\partial \theta} = -\frac{1}{\theta} + \frac{y}{\theta^2}$$

and

$$\frac{\partial^2 \ln[f(y|\theta)]}{\partial \theta^2} = \frac{1}{\theta^2} - \frac{2y}{\theta^3}.$$

Thus

$$E\left[\frac{-\partial^2 \ln[f(Y|\theta)]}{\partial \theta^2}\right] = -\frac{1}{\theta^2} + \frac{2E(Y)}{\theta^3} = -\frac{1}{\theta^2} + \frac{2\theta}{\theta^3} = \frac{1}{\theta^2}.$$

Then an approximate large-sample $100(1-\alpha)\%$ confidence interval for $\theta^2$ is

$$t(\hat{\theta}) \pm z_{\alpha/2}\sqrt{\left(\frac{\left(\frac{\partial t(\theta)}{\partial \theta}\right)^2}{nE\left[\frac{-\partial^2 \ln f(Y|\theta)}{\partial \theta^2}\right]}\right)\Bigg|_{\theta=\hat{\theta}}} = \overline{Y}^2 \pm z_{\alpha/2}\sqrt{\left(\frac{(2\theta)^2}{n\left(\frac{1}{\theta^2}\right)}\right)\Bigg|_{\theta=\overline{Y}}}$$

$$= \overline{Y}^2 \pm z_{\alpha/2}\sqrt{\frac{4\overline{Y}^4}{n}}$$

$$= \overline{Y} \pm z_{\alpha/2}\left(\frac{2\overline{Y}^2}{\sqrt{n}}\right)$$

**10.2** The test statistic $Y$ has a binomial distribution with $n = 20$ and $p$.

**a.** A Type I error occurs if the experimenter concluded that the drug dosage level induces sleep in less than 80% of the people suffering from insomnia when, in fact, drug dosage level does induce sleep in 80% of insomniacs.

**b.** $\alpha = P(\text{reject } H_0 | H_0 \text{ true}) = P(Y \leq 12 | p = .8) = .032$, using Table 1, Appendix III.

**c.** A Type II error would occur if the experimenter concluded that the drug dosage level induces sleep in 80% of the people suffering from insomnia when, in fact, fewer than 80% experience relief.

**d.** If $p = .6$,
$\beta = P(\text{accept } H_0 | H_0 \text{ false}) = P(Y > 12 | p = .6) = 1 - P(Y \leq 12 p = .6) = 1 - .584$
$= .416$

**e.** If $p = .4$, then
$\beta = P(Y > 12 | p = .4) = 1 - P(Y \leq 12 | p = .4) = 1 - .979 = .021.$

**10.4** **a.** A Type I error occurs if we conclude that the proportion of ledger sheets with errors is larger than .05 when, in fact, the proportion is .05.

**b.** By the scheme being used, we will reject for the following situations:
(NOTE: $NE$ = no error, E = error)

| Sheet 1 | Sheet 2 | Sheet 3 |
|---|---|---|
| $NE$ | $NE$ | . |
| $NE$ | $E$ | $NE$ |
| $E$ | $NE$ | $NE$ |
| $E$ | $E$ | $NE$ |

Thus, $\alpha = (.95)^2 + 2(.05)(.95)^2 + (.05)^2(.95) = .9025 + .09025 + .002375 = .995125.$

**c.** A Type II error occurs if we conclude that the proportion of ledger sheets with errors is .05 when, in fact, the proportion is larger than .05.

**d.** $\beta = P(\text{accept } H_0 \text{ when } H_a \text{ is true}) = P(\text{accepting } H_0 | p = p_a)$
$= 2p_a^2(1 - p_a) + p_a^3$. Since we reject if we observe E,E,E or NE,E,E or E,NE,E,