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Impact of measurement error on container inspection policies at port-of-entry

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Abstract Containers arriving at a port-of-entry are inspected using sensors and devices to detect drugs, weapons, nuclear materials and other illegal cargo. Measurement errors associated with the inspection process may result in higher misclassification of containers. In this paper, we propose and formulate three inspection policies for containers at port-of-entry assuming the presence of sensor measurement errors. The optimization of the policies is carried out and the performance of each in terms of misclassification probabilities is compared. In each of the policies, the optimum settings are determined by minimizing the probability of false rejection while limiting the probability of false acceptance to a specified tolerance level. The results show that repeat inspections improve performance in terms of correct container classification. Expressions are presented for container misclassification in a single station, as well as in systems with several inspection stations arranged in different configurations such as series, parallel, series-parallel and parallel-series.

Keywords POE · Container inspection · Measurement error · Sensor threshold · Re-inspection band · Container misclassification

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1 Introduction

Global commerce is totally dependent on the movement of shipping containers, which carry about 95 percent of the world's international cargo in terms of value. Containers carry a wide range of materials, food, equipment and other types of products and commodities. They may also transport drugs, arms, chemical, nuclear and biological materials and operatives for illegal activities, yet fewer than two percent of them are subject to in-depth inspection (Dahlman et al. 2005).

Disruption of maritime shipping due to risk associated with its contents would have a profound effect on the world economy. Moreover, the increasing dependence of companies on just-in-time deliveries of raw material and components and the global distribution of products and extensive supply chain networks between suppliers and companies magnify the impact of security issues in maritime shipping. This has prompted the investigation and implementation of different procedures for ensuring container security. The Department of Homeland Security recognizes that the risk associated with seaborne containers bound for the United States begins at the point of origin, which should also be the point of inspection. Beginning U.S. control over cargo at the foreign point of origin creates a "virtual border," a multi-layered defense addressing container security from the initial loading of the container to its movement through the entire international transportation network. The concept of a virtual border forms the foundation for the Container Security Initiative adopted by the United States bilaterally with a number of other countries (USCBP 2006). In addition to assessing the container risk at its origin port, the inspection of containers at ports-of-entry (POE) is critical for detecting and preventing illegal cargo from entering the United States. The inspection process can be generalized as the collection and analysis of information attained from the application of different types of inspection to decide whether to allow a container to pass through the port. Inspection may be in the form of radiation, biological and chemical detectors, other types of sensor instruments, or even manual inspection. Clearly, the accuracy of inspection in terms of passing (or accepting) those containers which do not have illegal cargo with minimum delays and rejecting those which indeed contain such cargo depends on many factors. These include the acceptance threshold levels and precision of the instruments or sensors used as well as the operational policies in place. Therefore, formulating a mathematical model of the container inspection allows for evaluation and improvement of the process. The POE inspection process has been investigated by Elsayed et al. (2009), Boros et al. (2009), Ramirez-Marquez (2008) and Wein et al. (2006). These papers consider the inspection process as a sequence of sensors (instruments or equipment), where each is dedicated to the detection of a specific characteristic of the undesirable material in the container. For instance, gamma-ray is used for the detection of radioactive material, while x-ray imaging is used for the analysis of images of unusual or unexpected contents and biological instruments are used to detect biological agents. The models investigated focus on the optimization of the inspection process by determining the threshold levels and sequence of inspection stations. The terms inspection station, sensor, and device are used interchangeably in this paper. A collection of stations forms an inspection system.

Brandenstein (2007) provides highlights of the role risk assessment played in the United States technology program for nonintrusive inspection of cargo containers for illicit drugs. Koch (2007) develops a port simulation model to investigate the effect of the introduction of new inspection technologies on the overall port operations.

In modeling POE inspection systems, the investigators seek the optimum threshold levels of the specific container characteristics (such as acceptable radiation level) and the optimum sequence of inspection that minimize the total cost or inspection time in the system. Dye (2003) summarizes some basic requirements of inspection systems: 1. Sensor systems must be operationally practical and must provide information that can enable effective, preemptive actions to be taken; 2. Sensor systems must be highly sensitive, providing a low probability of missed detections (false negatives) and 3. Sensor systems must give a low probability of false alarms (false positives).

No sensor system can provide a "perfect" solution to these competing requirements; the best that can be done is a compromise that strikes a balance among all three. Such a compromise may entail a layered defense, exploiting a combination of complementary sensor types. By sensing different characteristics at successive layers, the system-wide count of false negatives is greatly reduced, as one sensor's strengths can be used to offset another's weaknesses. Likewise, successive layers help reduce false positives. Another approach is to minimize the impact of measurement errors resulting from the sensor equipment in use. Large measurement errors may result in significant misclassifications of the containers (false positive and false negative). Therefore, one needs to minimize the impact of such errors (Mader et al. 1999) on the accuracy of container classifications. This can be achieved by considering the measurement errors in the inspection model and/or the development of effective inspection policies. Human influence, sensors and environmental factors contribute to measurement errors. It is important that the sources are identified and their contributions are reduced. In many cases it is difficult, if not impossible, to do so. However, researchers have utilized two approaches in order to minimize the effect of measurement errors (Kim et al. 2007). The first approach deals with the reduction of variability in measurements through the use of more precise measurement devices and/or better-trained operators (Chandra and Schall 1988; Chen and Chung 1996; Tang and Schneider 1988). The second approach is based on the use of guard bands in identifying "good" and "bad" items. The economic impact of guard bands is investigated by Eagle (1954), Grubbs and Coon (1954) and Hutchinson (1991). Deaver (1995) provides a comparative study of several strategies for the use of guard bands. McCarville and Montgomery (1996) develop an experimental design approach for finding the optimal guard bands for serial gauges. Recently, Kim et al. (2007) integrate these two approaches and develop an optimization scheme for the design of the most economical measurement procedures that simultaneously determine both the optimum precision level and guard band in order to reduce the impact of measurement errors. This paper considers approaches to reduce the impact of measurement errors in POE inspection policies. As mentioned, Elsayed et al. (2009) develop a model of the inspection system and illustrate how an inspection policy can be optimized. However, the model uses a single variable to represent the sensor reading and does not consider sensor measurement errors independently from the natural variation in the container attribute values. The measurement errors associated with inspection devices have a significant effect on the inspection decisions, and taking this into account would improve the model's accuracy. We consider realistic situations where measurement errors exist and are embedded in the readings obtained by the inspection devices. When a simple accept/reject threshold is used, containers with readings close to the threshold value are at risk for misclassification. Therefore, we investigate and optimize container inspection policies under different inspection strategies involving repeat inspections.

The paper is organized as follows. Section 2 describes the port-of-entry inspection problem and an associated measurement error model. This section also introduces three inspection policies and corresponding optimization problems. Section 3 presents the mathematical formulation of the inspection policies for a single station. Section 4 extends each policy from a single inspection station to systems where the inspection stations are configured in series, parallel, series-parallel and parallel-series arrangements. Section 5 presents numerical examples of a single station as well as different system configurations to evaluate the performance of each policy. The last section offers conclusions and discussion of the present work.

2 Problem description

2.1 Container inspection

Containers arriving to a port-of-entry are inspected to prevent entry of undesired cargo such as illegal weapons, drugs and dangerous materials. Each container has several attributes associated with undesired cargo and the detection of one or more of the attributes may lead to additional inspection such as examining the contents of the container manually. Attributes may include indicators of the presence of radioactive material, biological and chemical agents, drugs and illegal weapons. A typical inspection system may consist of several stations; each inspects one particular attribute of a container. Elsayed et al. (2009) consider an inspection system as a collection of n stations where inspection for the attributes in a given container is performed sequentially. At each station the detection of an attribute is dependent on preset threshold levels corresponding to that attribute. The overall decision to accept or reject a container is based on the outcome of individual stations. Elsayed et al. (2009) simultaneously determine the optimum sequence of inspection or the structure of the inspection decision trees and the optimum thresholds of the inspection stations that minimize the total inspection cost. In this paper, we follow the same inspection process and consider a more realistic situation where the sensor or station readings include measurement errors. We begin with a simple model and consider one station that inspects only one attribute. A preset threshold level T is used: if $y > T$ then the container fails inspection and if $y \leq T$ then it passes, where y is the inspection reading of an attribute. It is clear that the station decision is dependent on the preset T and the measurement errors. We then extend the work to inspection systems with multiple stations operating under different configurations.

2.2 Sensor modeling with measurement errors

Let x represent the true status of a particular container attribute, such that a container with $x = 0$ is acceptable and one with $x = 1$ is unacceptable. Over the entire container population the probability of $x = 1$ is $P(x = 1) = \pi$ and the probability of $x = 0$ is $P(x = 0) = 1 - \pi$. Since the true attribute reading y is dependent on x, following Stroud and Saeger (2003) and Elsayed et al. (2009), we assume two different distributions: $(y|x = 0) \sim N(\mu_0, \sigma_0^2)$ and $(y|x = 1) \sim N(\mu_1, \sigma_1^2)$. We choose to use the normal distribution because normally distributed data are the most commonly seen in practice and this assumption has been used in port-of-entry inspection applications, such as Stroud and Saeger (2003), Elsayed et al. (2009) and others. As mentioned in Elsayed et al. (2009), the development can also be extended to cover some other distributions. In this paper, we further assume that the measurement r taken by a sensor is affected by both the true attribute value y and some random measurement errors ε, so that $r = y + \varepsilon$. Here y and ε are independent. So, if we assume the measurement errors are distributed normally, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, then the distributions of observed readings from a sensor can be written as $(r|x = 0) \sim N(\mu_0, \sigma_0^2 + \sigma_\varepsilon^2)$ and $(r|x = 1) \sim N(\mu_1, \sigma_1^2 + \sigma_\varepsilon^2)$.

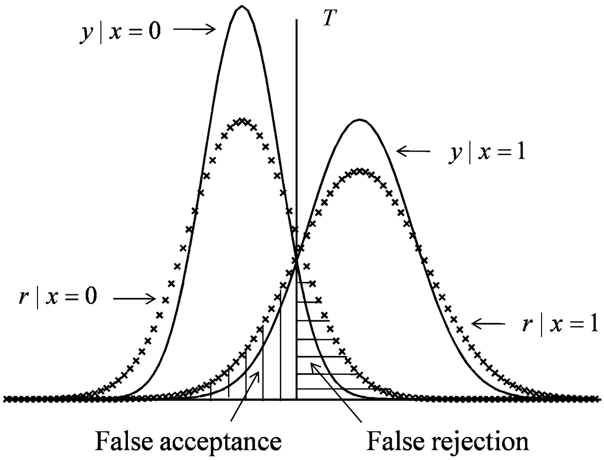

Fig. 1 Probabilities of false acceptance and false rejection

When a simple threshold is applied to make a decision, there is a chance to accept an unacceptable container or reject an acceptable one, called misclassification errors. Consider the simple inspection process mentioned in Sect. 2.1, in which a sensor reading r, which includes some random measurement error in addition to the true attribute value y, is compared to a specified threshold value T, returning an "accept" decision $d = 0$ if $r \leq T$ and a "fail" decision $d = 1$ if $r > T$. In this decision making process, there is a chance to reject an acceptable container (with true status $x = 0$). Such a misclassification error is the probability of false rejection (*PFR*). There is also a chance to accept an unacceptable container (with true status $x = 1$). Such an error is the probability of false acceptance (*PFA*).

The presence of measurement errors increases both the variability of the measurements and misclassification of containers in the inspection process. Figure 1 shows the effect of measurement error on the *PFR* and *PFA* values of an inspection system. The probability of false rejection corresponds to the area to the right of T and under the curve that represents the measurement values for $x = 0$. Comparing the dotted and solid lines which represent $r|x = 0$ and $y|x = 0$, with and without measurement errors, respectively, it is clear that the area representing *PFR* for $r|x = 0$, which is highlighted with horizontal shading, is greater than that for $y|x = 0$. This difference corresponds to the increase in *PFR* when measurement errors are included in the sensor reading r. Similarly, the area to the left of T under the curve that represents measurement values for $x = 1$ corresponds to the probability of false acceptance. The area under the dotted line for $r|x = 1$ is highlighted by vertical shading and is clearly greater than the area under the solid line for $y|x = 1$. This signifies that the *PFA* in the presence of measurement errors associated with the sensor reading r is greater than the *PFA* in the absence of measurement errors.

2.3 Inspection policies

One way to reduce the impact of sensor measurement errors on misclassification is to repeat measurements with the same sensor. Multiple measurements (r values) taken by a given sensor have the same y value but different ε values and since y and ε are independent, averaging these measurements provides a more accurate estimate compared to a single measurement. In light of this, inspection policies can be developed that require repeated measurements (referred to as repeat inspection) for selected containers. The method for selecting containers for repeat inspection should identify ambiguous containers at higher risk for misclassification. This can be achieved by applying a "re-inspection band" b around T so that containers with $T - b/2 \le r \le T + b/2$ are subject to additional inspections. The concept for using a re-inspection band in the selection of containers for repeat inspection is related to the use of guard bands. However, different from the use of guard bands in the quality control literature, we do not accept containers within the band limits in the POE inspection practice but they are rather selected for further inspection. These ideas are utilized in formulating the policies described below.

Policy I is a simple inspection process of comparing sensor readings to a preset threshold level. This policy is not adjusted for errors inherent in the measurements. Specifically, a measurement value r is obtained from a sensor and containers with $r \le T$ are accepted, while those with $r > T$ are rejected. The value of T which is given as part of the policy has a significant effect on the performance of the inspection station and therefore is used as a decision variable in the optimization of Policy I. This policy is simple and inexpensive to apply, however it is not adjusted for measurement errors inherent in the measurements. The presence of measurement errors can have a significant effect on the performance of the inspection process, as illustrated in Fig. 1. Although unavoidable in many cases, random measurement errors are often ignored in theory and practice due to a variety of reasons.

In manufacturing and production environments, to satisfy certain quality control criterion and also to avoid the high penalty cost of mistakenly accepting non-conforming units, units are often subjected to 100 percent repeat inspection. This can reduce the effects of measurement error in terms of *PFA* and *PFR*. However, such frequent use of repeat inspection in the port-of-entry problem is likely to increase waiting time and delay the delivery of containers. From Fig. 1, containers with readings close to the threshold T are at high risk for misclassification and applying repeat inspection to those containers reduces the probability of misclassification. Therefore, repeat inspection is conducted in the following policies using a re-inspection band of width b around the threshold T, which can both be adjusted for optimal performance. We integrate the re-inspection band and repeat inspection into two inspection policies to reduce the impact of measurement errors on misclassification rates.

Policy II is an example of using repeat inspection to decrease the measurement errors when a more precise inspection sensor is unavailable. The first step of this two-step process is to apply a re-inspection band b to create a range around T and assess measurement values from an initial inspection so that containers with readings falling above or below these limits are rejected or accepted, respectively. Figure 2 illustrates the effect of the re-inspection band on *PFR* and *PFA* in decisions made in the first step of Policy II. Comparing Figs. 1 and 2, it is obvious that the *PFA* and *PFR* of those decisions falling outside the re-inspection band applied around T are less than the total *PFA* and *PFR* of Policy I.

Containers with initial readings falling within $T - b/2 \le r \le T + b/2$ are selected for the second step of Policy II, in which the inspection is repeated n times with the same sensor and the average reading is used to provide a more accurate measurement. This average is then compared to T for a final accept/reject decision. The decision variables in this policy are b and T.

Taking the average of n repeat measurements reduces the variability of the readings. An alternative approach to reduce measurement error is to select sensors with higher precision as described in Policy III below. Although it may depend on parameter choices, Policy III is typically capable of improving performance over Policy II, since it uses more precise sensors in taking subsequent measurements, which can reduce the impact of measurement errors. It should be noted that an inspection process with higher precision usually incurs higher inspection cost due to the requirement of advanced equipment, better trained operators and well controlled environment, etc.

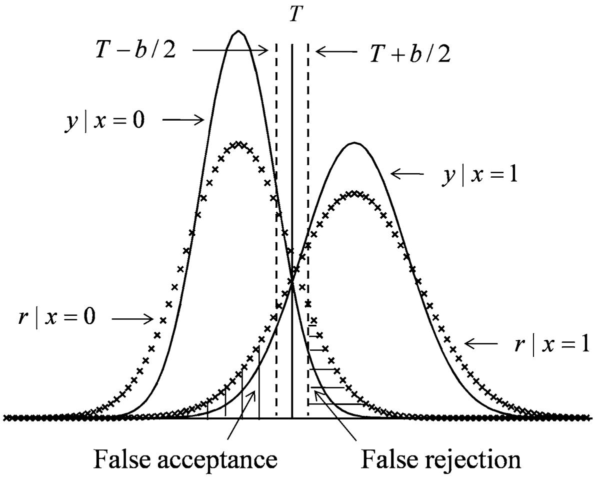

Fig. 2 Re-inspection band and misclassification errors

In forming Policy III, we assume that a series of measurements can be obtained, each with improved accuracy. This could be achieved either by adjusting the operational settings of one sensor, using different sensors, or some combination of the two. Policy III has the same first step as Policy II, where initial readings are compared with the re-inspection band to make decisions to accept, re-inspect when the readings are within the band, or reject containers. The repeat inspection is done using a sensor with improved precision, $\sigma_{R\varepsilon}^2 < \sigma_\varepsilon^2$. This process of inspection using a more precise sensor and applying a re-inspection band can be repeated n times, at which time a final measurement is taken and compared directly with T to make an accept/reject decision, as in Policy I.

To find the optimal settings for these policies, we must first define an objective function. The probability of falsely clearing dangerous cargo (*PFA*) and the rate of false detection (*PFR*) are useful measures of inspection performance. One possible objective is to minimize *PFA* while constraining the maximum acceptable *PFR*. In this paper, we define the objective function for all policies to be the minimization of *PFR* and specify FA^* as an upper limit for *PFA*. Thus the optimization problem can be written as:

$$\begin{array}{ll} \min & PFR \\ \text{s.t.} & PFA \le FA^* \end{array}$$

The expressions for *PFR* and *PFA* and decision variables for each policy in a single station are derived in the following section. Expansions for systems with multiple stations are presented in Sect. 4.

3 Formulation of inspection policies in a single station

3.1 Policy I

Policy I is a simple inspection process where each sensor reading r is compared against a preset decision threshold T to make an accept or reject decision: if $r > T$, a reject decision $d = 1$ is made, otherwise an accept decision $d = 0$ is made. According to the additive error model, $r = y + \varepsilon$, and assumptions of the distribution of measurement readings, $(r|x = 0) \sim N(\mu_0, \sigma_0^2 + \sigma_\varepsilon^2)$ and $(r|x = 1) \sim N(\mu_1, \sigma_1^2 + \sigma_\varepsilon^2)$ for acceptable and unacceptable containers respectively, the *PFR* and *PFA* for Policy I are given by:

$$PFR = P(d = 1|x = 0) = 1 - \Phi\left(\frac{T - \mu_0}{\sqrt{\sigma_0^2 + \sigma_\varepsilon^2}}\right),$$

$$PFA = P(d = 0|x = 1) = \Phi\left(\frac{T - \mu_1}{\sqrt{\sigma_1^2 + \sigma_\varepsilon^2}}\right),$$

where Φ denotes the standard cumulative normal distribution.

The optimization of Policy I is defined as minimizing the probability of false rejection while limiting the probability of false acceptance at some low level, which is often defined by a requirement. This optimization is formulated as $\min PFR$ subject to $PFA \le FA^*$. The decision variable for this policy is T.

3.2 Policy II

Policy II is a two step process which employs a re-inspection band and repeat inspections in an attempt to reduce the impact of measurement errors. In the first step, the inspection reading r_1 of an attribute is obtained and compared against the limits of the re-inspection band b placed around a given T. Containers with $r_1 < T - b/2$ are accepted and containers with $r_1 > T + b/2$ are rejected. The remaining containers with r_1 values that fall within the re-inspection band ($T - b/2 \le r_1 \le T + b/2$) proceed to the next step, in which they are subjected to $l - 1$ repeat inspections with the same sensor. We assume the total number of inspections l is given. Let $r_2, \dots, r_l$ be the observed values from the $l - 1$ repeat inspections, and let $\bar{r}_{2:l}$ be their average value. If $\bar{r}_{2:l} \le T$, the container is accepted and if $\bar{r}_{2:l} > T$ the container is rejected. The distribution of the average reading $\bar{r}_{2:l}$ is given by $\bar{r}_{2:l}|x = i \sim N(\mu_i, \sigma_i^2 + \sigma_\varepsilon^2/(l - 1))$ for $i = 0, 1$. Note that $r_1 - y|x = i \sim N(0, \sigma_\varepsilon^2)$ and $\bar{r}_{2:l} - y|x = i \sim N(0, \sigma_\varepsilon^2/(l - 1))$ for $i = 0, 1$.

The total *PFR* for Policy II is

$$\begin{aligned}
&P\{D = 1|x = 0\} \\
&\quad = P\{d_1 = 1|x = 0\} + P\left\{d_{2:l} = 1|x = 0, T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\right\} \\
&\qquad \times P\left\{T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\middle|x = 0\right\} \\
&\quad = P\left\{r_1 > T + \frac{b}{2}\middle|x = 0\right\} \\
&\qquad + \int_{-\infty}^{\infty} P\left\{\bar{r}_{2:l} > T|x = 0, y, T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\right\} \\
&\qquad \times P\left\{T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\middle|x = 0, y\right\} f(y)dy \\
&\quad = P\left\{r_1 > T + \frac{b}{2}\middle|x = 0\right\} \\
&\qquad + \int_{-\infty}^{\infty} P\left\{\bar{r}_{2:l} - y > T - y|x = 0, T - \frac{b}{2} \le r_1 \le T + \frac{b}{2}\right\}
\end{aligned}$$

$$
\begin{aligned}
&\times P\left\{T-\frac{b}{2}-y \le r_1 - y \le T+\frac{b}{2}-y|x=0\right\} f(y)dy \\
&= 1-\Phi\left(\frac{T+\frac{b}{2}-\mu_0}{\sqrt{\sigma_0^2+\sigma_\varepsilon^2}}\right) \\
&\quad + \int_{-\infty}^{\infty}\left\{1-\Phi\left(\frac{T-y}{\sigma_\varepsilon/\sqrt{l-1}}\right)\right\}\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_\varepsilon}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_\varepsilon}\right)\right\}\frac{\phi(\frac{y-\mu_0}{\sigma_0})}{\sigma_0}dy
\end{aligned}
$$

and the total *PFA* for Policy II is

$$
\begin{aligned}
&P\{D=0|x=1\} \\
&\quad = P\{d_1=0|x=1\} + P\left\{d_{2:l}=0|x=1, T-\frac{b}{2}\le r_1 \le T+\frac{b}{2}\right\} \\
&\qquad \times P\left\{T-\frac{b}{2}\le r_1\le T+\frac{b}{2}\middle|x=1\right\} \\
&\quad = P\left\{r_1 < T-\frac{b}{2}\middle|x=1\right\} \\
&\qquad + \int_{-\infty}^{\infty} P\left\{\bar{r}_{2:l} < T|x=1, y, T-\frac{b}{2}\le r_1\le T+\frac{b}{2}\right\} \\
&\qquad \times P\left\{T-\frac{b}{2}\le r_1 \le T+\frac{b}{2}\middle|x=1,y\right\} f(y)dy \\
&\quad = P\left\{r_1 < T-\frac{b}{2}\middle|x=1\right\} \\
&\qquad + \int_{-\infty}^{\infty} P\left\{\bar{r}_{2:l}-y < T-y|x=1, T-\frac{b}{2}\le r_1\le T+\frac{b}{2}\right\} \\
&\qquad \times P\left\{T-\frac{b}{2}-y\le r_1-y\le T+\frac{b}{2}-y|x=1\right\} f(y)dy \\
&\quad = \Phi\left(\frac{T-\frac{b}{2}-\mu_1}{\sqrt{\sigma_1^2+\sigma_\varepsilon^2}}\right) \\
&\qquad + \int_{-\infty}^{\infty}\Phi\left(\frac{T-y}{\sigma_\varepsilon/\sqrt{l-1}}\right)\left\{\Phi\left(\frac{T+\frac{b}{2}-y}{\sigma_\varepsilon}\right)-\Phi\left(\frac{T-\frac{b}{2}-y}{\sigma_\varepsilon}\right)\right\}\frac{\phi(\frac{y-\mu_1}{\sigma_1})}{\sigma_1}dy,
\end{aligned}
$$

where ϕ denotes the probability density function of standard normal distribution, D denotes the final decision, d_1 denotes the decision in the first inspection and $d_{2:l}$ denotes the decision after re-inspection in the second step. The optimization problem of Policy II is again defined as $\min PFR$ subject to $PFA \le FA^*$. The decision variables here are T and b.

3.3 Policy III

Policy III is a multiple stage process that employs sensors of increasing precision levels to decrease measurement error in subsequent inspections. In the first step, the inspection reading r_1 is compared against the limits of a re-inspection band b_1 placed around a given T. Let the variance of measurement errors associated with the first inspection be designated as $\sigma^2_{\varepsilon,1}$. Containers with $r_1 < T - b_1/2$ are accepted and containers with $r_1 > T + b_1/2$ are rejected. The remaining containers with r_1 values that fall within the re-inspection band $(T - b_1/2 \le r_1 \le T + b_1/2)$ proceed to the next step where they are inspected using a sensor with a higher level of precision which returns an observed value r_2. This second inspection benefits from a smaller variance in measurement errors, $\sigma^2_{\varepsilon,2} < \sigma^2_{\varepsilon,1}$ corresponding to the higher precision level, and the observed values are compared against a re-inspection band b_2 around T. If $r_2 < T - b_2/2$, the container is accepted; if $r_2 > T + b_2/2$, the container is rejected. If $T - b_2/2 \le r_2 \le T + b_2/2$, the container is then inspected by yet a more precise sensor with smaller variance in measurement errors, $\sigma^2_{\varepsilon,3} < \sigma^2_{\varepsilon,2}$ and the observed value from this third inspection is designated r_3. This process of taking readings and making decision in three ways (accept, reject, re-inspect) is repeated $l - 1$ times (l is given) to get $r_1, \dots, r_{l-1}$ for given $b_1, \dots, b_{l-1}$. In the last step, the lth inspection is performed and the reading is compared against T; if $r_l \le T$ the container is accepted and if $r_l > T$ the container is rejected. Note that, in each step, we assume that the sensor is more precise than that in the previous step, i.e., $\sigma^2_{\varepsilon,k} < \sigma^2_{\varepsilon,k-1}$ for $k = 2, \dots, l$.

For Policy III, the total *PFR* over l inspections is

$$
\begin{aligned}
&P\{D=1|x=0\}\\
&\quad = P\{d_1 = 1|x=0\}\\
&\qquad + \sum_{k=2}^{l} P\left\{d_k = 1|x=0, T - \frac{b_j}{2} \le r_j \le T + \frac{b_j}{2}, 1 \le j < k\right\}\\
&\qquad \times P\left\{T - \frac{b_j}{2} \le r_j \le T + \frac{b_j}{2}, 1 \le j < k|x=0\right\}\\
&\quad = 1 - \Phi\left(\frac{T + \frac{b_1}{2} - \mu_0}{\sqrt{\sigma_0^2 + \sigma^2_{\varepsilon,1}}}\right)\\
&\qquad + \sum_{k=2}^{l-1} \int_{-\infty}^{\infty} \left\{1 - \Phi\left(\frac{T + \frac{b_k}{2} - y}{\sigma_{\varepsilon,k}}\right)\right\}\\
&\qquad \times \prod_{j=1}^{k-1} \left\{\Phi\left(\frac{T + \frac{b_j}{2} - y}{\sigma_{\varepsilon,j}}\right) - \Phi\left(\frac{T - \frac{b_j}{2} - y}{\sigma_{\varepsilon,j}}\right)\right\} \frac{\phi(\frac{y-\mu_0}{\sigma_0})}{\sigma_0} dy\\
&\qquad + \int_{-\infty}^{\infty} \left\{1 - \Phi\left(\frac{T - y}{\sigma_{\varepsilon,l}}\right)\right\}\\
&\qquad \times \prod_{j=1}^{l-1} \left\{\Phi\left(\frac{T + \frac{b_j}{2} - y}{\sigma_{\varepsilon,j}}\right) - \Phi\left(\frac{T - \frac{b_j}{2} - y}{\sigma_{\varepsilon,j}}\right)\right\} \frac{\phi(\frac{y-\mu_0}{\sigma_0})}{\sigma_0} dy
\end{aligned}
$$

and the total *PFA* over l inspections is

$$
\begin{aligned}
&P\{D=0|x=1\}\\
&\quad = P\{d_1 = 0|x=1\}
\end{aligned}
$$

$$+\sum_{k=2}^{l} P\left\{d_k = 0|x=1, T-\frac{b_j}{2} \le r_j \le T+\frac{b_j}{2}, 1 \le j < k\right\}$$

$$\times P\left\{T-\frac{b_j}{2} \le r_j \le T+\frac{b_j}{2}, 1 \le j < k|x=1\right\}$$

$$= \Phi\left(\frac{T-\frac{b_1}{2}-\mu_1}{\sqrt{\sigma_1^2+\sigma_{\varepsilon,1}^2}}\right)$$

$$+\sum_{k=2}^{l-1}\int_{-\infty}^{\infty}\Phi\left(\frac{T-\frac{b_k}{2}-y}{\sigma_{\varepsilon,k}}\right)\prod_{j=1}^{k-1}\left\{\Phi\left(\frac{T+\frac{b_j}{2}-y}{\sigma_{\varepsilon,j}}\right)-\Phi\left(\frac{T-\frac{b_j}{2}-y}{\sigma_{\varepsilon,j}}\right)\right\}\frac{\phi(\frac{y-\mu_1}{\sigma_1})}{\sigma_1}dy$$

$$+\int_{-\infty}^{\infty}\Phi\left(\frac{T-y}{\sigma_{\varepsilon,l}}\right)\prod_{j=1}^{l-1}\left\{\Phi\left(\frac{T+\frac{b_j}{2}-y}{\sigma_{\varepsilon,j}}\right)-\Phi\left(\frac{T-\frac{b_j}{2}-y}{\sigma_{\varepsilon,j}}\right)\right\}\frac{\phi(\frac{y-\mu_1}{\sigma_1})}{\sigma_1}dy,$$

where D denotes the final decision and d_k denotes the decision following the kth inspection for $k = 1, \dots, l$. Similar to Policy I and II, the optimization of Policy III is defined as $\min PFR$ subject to $PFA \le FA^*$. Assuming the sequence of $\sigma_{\varepsilon,k}^2$ values is given, the decision variables in this policy are T and $b_1, \dots, b_{l-1}$.

4 Modeling container inspection systems

At a port-of-entry, containers arriving for inspection are either inherently acceptable or contain unacceptable materials, and they have several attributes which may reflect the presence or absence of such materials. If we assume one sensor inspects one specific attribute and returns an acceptance-or-rejection decision (0 or 1 respectively), the container inspection problem of examining multiple attributes can then be viewed from a system level. Since each sensor is subject to measurement error, the modeling of a single sensor inspection process considering measurement error can be extended to the system level that considers all attributes of a container.

An inspection system collects decisions from individual sensors and classifies a container based on a system decision function F that assigns to each binary string of decisions $(D_1, D_2, \dots, D_n)$ an overall decision $D = F(D_1, D_2, \dots, D_n)$ of 0 or 1 (accept or reject, respectively). The system decision function F does not necessarily represent the physical configuration or layout of the inspection stations but rather the logical flow incorporating station decisions. In this paper we define, for instance, a system that uses a series Boolean function as applying a decision function F that assigns the container class "1" if any of the individual decisions are "fail", $D_i = 1$ for any station i. In this case, $F(D_1, D_2, \dots, D_n) = (D_1 \vee D_2 \vee \cdots D_n)$. A system that uses a parallel Boolean function is defined as applying a decision function F that assigns the container class "1" only if all of the individual decisions are "fail". In this case, $F(D_1, D_2, \dots, D_n) = (D_1 \wedge D_2 \wedge \cdots D_n)$. Elsayed et al. (2009) define series, parallel, series-parallel and parallel-series Boolean decision functions for an inspection system. These and other Boolean decision functions can be applied in our work to integrate the decisions from individual inspection stations into the final decision regarding the acceptance of a container. For illustrative purposes we present the extension from a single station to an inspection decision system based on these four Boolean decision functions.

Let $(x_1 x_2 \cdots x_n)$ represent the true statuses of independent container attributes, such that $x_i = 0$ indicates the container is acceptable with respect to attribute i, and $x_i = 1$ indicates the presence of undesired cargo related to attribute i. It is assumed that the acceptability of a container is defined by a function of the individual container attributes, $F(x_1 x_2 \cdots x_n)$ where F is the same function as the one introduced in the previous paragraph to make the overall system decision. In the overall container population $P(x_i = 1) = \pi_i$ and $P(x_i = 0) = 1 - \pi_i$. The measurement of each attribute depends on the true status of the attribute, thus we assume two distributions: $(y_i | x_i = 0) \sim N(\mu_{0i}, \sigma_{0,i}^2)$ and $(y_i | x_i = 1) \sim N(\mu_{1i}, \sigma_{1,i}^2)$ at station i. The random measurement error of each sensor is assumed to be normally distributed as $\varepsilon_i \sim N(0, \sigma_{\varepsilon,i}^2)$. The error term is independent of x_i. By the additive error model, $r_i = y_i + \varepsilon_i$, the observed readings from sensor i can be written as $(r_i | x_i = 0) \sim N(\mu_{0i}, \sigma_{0,i}^2 + \sigma_{\varepsilon,i}^2)$ and $(r_i | x_i = 1) \sim N(\mu_{1i}, \sigma_{1,i}^2 + \sigma_{\varepsilon,i}^2)$.

The inspection policies developed in the previous sections can be extended to inspection systems. To simplify the presentation, we assume that all the sensors in an inspection system use the same policy, even though, in practice, different policies may be applied to different sensors in the same inspection system.

4.1 Series Boolean decision function

The series decision function could be applied in a system where various risks may be indicated by different attributes, and the detection of any one of these attributes warrants further investigation. The decision function F for this type of system is defined above and referred to here as F_{series}. Then systematically, the probability of false rejection and false acceptance of a container are respectively obtained as

$$\begin{aligned} &PFR_{series} \\ &\quad = P[D=1 | F_{series}(X)=0] = \frac{P[D=1, F_{series}(X)=0]}{P[F_{series}(X)=0]} \\ &\quad = \frac{\sum_{F_{series}(x_1,\ldots,x_n)=0} \{P[D=1|X=(x_1,\ldots,x_n)]P[X=(x_1,\ldots,x_n)]\}}{\sum_{F_{series}(x_1,\ldots,x_n)=0} P[X=(x_1,\ldots,x_n)]} \\ &= \frac{P[D=1|X=(0,\ldots,0)]P[X=(0,\ldots,0)]}{P[X=(0,\ldots,0)]} = 1 - \prod_{i=1}^{n} P(D_i=0|x_i=0), \end{aligned} \tag{1}$$

$$\begin{aligned} &PFA_{series} \\ &\quad = P[D=0 | F_{series}(X)=1] = \frac{P[D=0, F_{series}(X)=1]}{P[F_{series}(X)=1]} \\ &\quad = \frac{\sum_{F_{series}(x_1,\ldots,x_n)=1} \{P[D=0|X=(x_1,\ldots,x_n)]P[X=(x_1,\ldots,x_n)]\}}{\sum_{F_{series}(x_1,\ldots,x_n)=1} P[X=(x_1,\ldots,x_n)]} \\ &\quad = \frac{\sum_{F_{series}(x_1,\ldots,x_n)=1} \{[\prod_{i=1}^{n} P(D_i=0|X_i=x_i)][\prod_{i=1}^{n} P(X_i=x_i)]\}}{1-\prod_{i=1}^{n}(1-\pi_i)}, \end{aligned} \tag{2}$$

where

$$P(X_i = x_i) = \begin{cases} \pi_i & \text{if } x_i = 1 \\ 1 - \pi_i & \text{if } x_i = 0. \end{cases} \tag{3}$$

The optimization problem of the system inspection policy can be formulated as minimize PFR subject to $PFA \le FA^*$. The system's PFR and PFA can be computed using (1) and (2), except that the expression of the corresponding $P(D_i = 0|X_i = x_i)$ is more complicated and is given in the Appendix for each policy. Note that in Policy I, the decision variables are the thresholds T_i. In Policies II and III, the decision variables are the thresholds T_i, as well the widths of the re-inspection bands b_i for station $i = 1, \ldots, n$ in Policy II, and $b_{i,j}$ for station $i = 1, \ldots, n$ and inspection step $j = 1, \ldots, l-1$ in Policy III.

4.2 Parallel Boolean decision function

The parallel decision function could be applicable in a system where each attribute is a partial indicator of a particular risk, and only the positive detection of every attribute would be considered significant evidence of unacceptability. The decision function for this type of system is defined above and referred to here as $F_{parallel}$. In a system using a parallel decision function, the probability of false rejection and false acceptance of a container are respectively obtained as

$$\begin{aligned}
&PFR_{parallel}\\
&\quad = \frac{\sum_{F_{parallel}(x_1,\ldots,x_n)=0}\{P[D=1|X=(x_1,\ldots,x_n)]P[X=(x_1,\ldots,x_n)]\}}{\sum_{F_{parallel}(x_1,\ldots,x_n)=0} P[X=(x_1,\ldots,x_n)]}\\
&\quad = \frac{\sum_{F_{parallel}(x_1,\ldots,x_n)=0}\{[\prod_{i=1}^n P(D_i=1|X_i=x_i)][\prod_{i=1}^n P(X_i=x_i)]\}}{1-\prod_{i=1}^n \pi_i},\\
&PFA_{parallel}\\
&\quad = \frac{\sum_{F_{parallel}(x_1,\ldots,x_n)=1}\{P[D=0|X=(x_1,\ldots,x_n)]P[X=(x_1,\ldots,x_n)]\}}{\sum_{F_{parallel}(x_1,\ldots,x_n)=1} P[X=(x_1,\ldots,x_n)]}\\
&\quad = \frac{P[D=0|X=(1,\ldots,1)]P[X=(1,\ldots,1)]}{P[X=(1,\ldots,1)]}\\
&\quad = 1-\prod_{i=1}^{n} P(D_i=1|X_i=1),
\end{aligned}$$

where $P(D_i = 1|X_i = x_i) = 1 - P(D_i = 0|X_i = x_i)$. Expressions for $P(D_i = 0|X_i = x_i)$ are given in the Appendix for each case where a sensor uses one of the three policies presented earlier in this paper.

4.3 Series-parallel Boolean decision function

Consider a series-parallel decision function that has n subsystems in series with m_i sensors in parallel in subsystem i. The decisions of individual station are denoted as D_{ij} for $i = 1, \ldots, n$ and $j = 1, \ldots, m_i$. A series-parallel decision function might be useful if each subsystem represents a different risk and a fail decision in any subsystem is significant, and within a particular subsystem each attribute is a partial indicator of that risk. The decision function for this type of system is referred to as $F_{series\text{-}parallel}$ and can be thought of as an F_{series} decision function of n elements, where each element is the result of an $F_{parallel}$ decision function for a subsystem of m_i sensors. In a system using a series-parallel decision function, the probability of false rejection and false acceptance of a container are respectively obtained as

$$PFR_{series\text{-}parallel} = \frac{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} \{P[D=1|X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]\}}{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]}$$

$$= \frac{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} \left\{\left[1-\prod_{i=1}^{n}\left(1-\prod_{j=1}^{m_i} P\left(D_{ij}=1|X_{ij}=x_{ij}\right)\right)\right]\left[\prod_{i=1}^{n}\prod_{j=1}^{m_i} P\left(X_{ij}=x_{ij}\right)\right]\right\}}{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]},$$

$$PFA_{series\text{-}parallel} = \frac{\sum_{F_{series]\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=1} \left\{P\left[D=0|X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i\right]P\left[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i\right]\right\}}{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=1} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]}$$

$$= \frac{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=1} \left\{\left[\prod_{i=1}^{n}\left(1-\prod_{j=1}^{m_i} P\left(D_{ij}=1|X_{ij}=x_{ij}\right)\right)\right]\left[\prod_{i=1}^{n}\prod_{j=1}^{m_i} P\left(X_{ij}=x_{ij}\right)\right]\right\}}{\sum_{F_{series\text{-}parallel}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=1} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]},$$

where $P(D_{ij}=1|X_{ij}=x_{ij}) = 1 - P(D_{ij}=0|X_{ij}=x_{ij})$. Expressions for $P(D_{ij}=0|X_{ij}=x_{ij})$ are given in the Appendix for each of the three policies presented in this paper, replacing the *ij* notation with a single subscript.

4.4 Parallel-series Boolean decision function

Consider a parallel-series decision function that has n parallel paths with m_i sensors in series in path i. The decisions of individual station are denoted as D_{ij} for $i = 1, \dots, n$ and $j = 1, \dots, m_i$. A parallel-series decision function might be useful if each path represents an indicator of one particular risk and only a "fail" decision in every path would signify the presence of that risk, and a positive indicator may arise from the detection of any one of the attributes within that particular path. The decision function for this type of system is referred to as $F_{parallel\text{-}series}$ and can be thought of as an $F_{parallel}$ decision function of n elements, where each element is the result of an F_{series} decision function for a path of m_i sensors. In a system using a parallel-series decision function, the probability of false rejection and false acceptance of a container are respectively obtained as

$$PFR_{parallel\text{-}series} = \frac{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} \{P[D=1|X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]\}}{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]}$$

$$= \frac{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} \left\{\left[\prod_{i=1}^{n}\left(1-\prod_{j=1}^{m_i} P\left(D_{ij}=0|X_{ij}=x_{ij}\right)\right)\right]\left[\prod_{i=1}^{n}\prod_{j=1}^{m_i} P\left(X_{ij}=x_{ij}\right)\right]\right\}}{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\dots,n;\\ j=1,\dots,m_i}\right)=0} P[X_{ij}=x_{ij}, i=1,\dots,n; j=1,\dots,m_i]},$$

$$PFA_{parallel\text{-}series} = \frac{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\ldots,n;\\ j=1,\ldots,m_i}\right)=1} \{P[D=0|X_{ij}=x_{ij}, i=1,\ldots,n; j=1,\ldots,m_i] P[X_{ij}=x_{ij}, i=1,\ldots,n; j=1,\ldots,m_i]\}}{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\ldots,n;\\ j=1,\ldots,m_i}\right)=1} P[X_{ij}=x_{ij}, i=1,\ldots,n; j=1,\ldots,m_i]}$$

$$= \frac{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\ldots,n;\\ j=1,\ldots,m_i}\right)=1} \left\{\left[1-\prod_{i=1}^{n}\left(1-\prod_{j=1}^{m_i} P\left(D_{ij}=0|X_{ij}=x_{ij}\right)\right)\right]\left[\prod_{i=1}^{n}\prod_{j=1}^{m_i} P\left(X_{ij}=x_{ij}\right)\right]\right\}}{\sum_{F_{parallel\text{-}series}\left(\substack{x_{ij}, i=1,\ldots,n;\\ j=1,\ldots,m_i}\right)=1} P[X_{ij}=x_{ij}, i=1,\ldots,n; j=1,\ldots,m_i]}.$$

Expressions for $P(D_{ij}=0|X_{ij}=x_{ij})$ are given in the Appendix for each of the three policies presented in this paper, replacing the *ij* notation with a single subscript.

The above configurations and the corresponding decision functions are well structured and can be generalized. However, the same methodology can also be used in non-structured configurations such as a network of sensors with either directed or non-directed flow of containers in the system. In all cases, it becomes important to clearly define the decision function that reflects the outcome of the inspection system. It is also important to note that the physical layout of the configuration does not necessarily need to match the logical decision function.

5 Numerical examples

5.1 Numerical example for the policies in a single station

A numerical example is presented in this subsection to demonstrate the effectiveness of the proposed inspection policies in a single station. Note that in the port-of-entry inspection problem, the goal is to separate two groups of containers and the problem is location and scale invariant (Elsayed et al. 2009). Without loss of generality, we choose $\mu_0 = 0$ and $\mu_1 = 1$ for all attribute distributions. In the numerical example, we use $\sigma_0 = 0.35$ and $\sigma_1 = 0.1$. Since in practice inspections can only be repeated a limited number of times, we assume that the total number of inspections for Policy II and III is limited to three, after which the container is either accepted or subjected to manual inspection. We also assume that the standard deviation of measurement errors in the initial inspection is $\sigma_\varepsilon = 0.06$. The standard deviations of measurement errors for the second and third inspections of Policy III are $\sigma_{\varepsilon,2} = 0.03$ and $\sigma_{\varepsilon,3} = 0.015$, respectively. Matlab is used to determine the optimal solution for all three policies by implementing a constrained nonlinear multivariable algorithm, *fmincon*. In the optimization, the upper and lower limits of the threshold value and re-inspection band width are limited between 0 and 1. The objective function is to minimize *PFR*, subject to a *PFA* constraint of 10^{-3}, 10^{-4} and 10^{-5}, respectively. The minimum *PFR* values with respect to the different *PFA* constraints for each policy are presented in Fig. 3. The associated optimal threshold and the band width are shown in Table 1. The optimal values of the two band widths of Policy III happen to be the same in this numerical example, therefore only one value is shown in the table.

It is evident from Fig. 3 that Policy II consistently returns a smaller *PFR* than Policy I, and Policy III consistently returns a smaller *PFR* than Policy II under each of the constraints. This indicates that the inspection process can be improved by the application of a re-inspection band and repeat inspections, and further improvement is possible if the repeat inspections are conducted by sensors with higher precision levels.

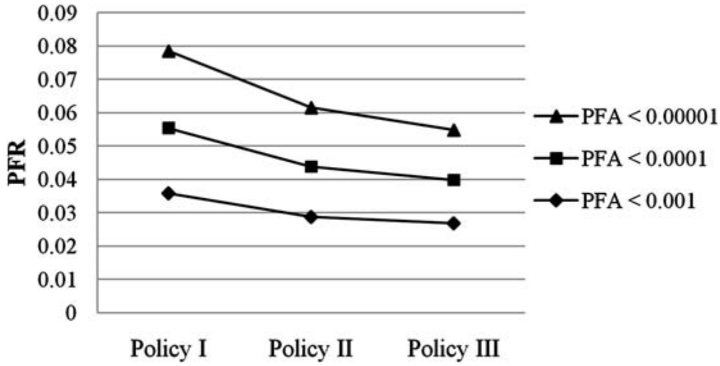

Fig. 3 Minimum *PFR* obtained from each policy subject to different constraints

Table 1 Optimal threshold value and band width associated with minimum *PFR*

*FA**	*PFR*				
	Policy I	Policy II		Policy III	
	Threshold	Band width	Threshold	Band width	Threshold
0.001	0.6396	0.1539	0.6689	0.4001	0.6875
0.0001	0.5663	0.1594	0.6015	0.3999	0.6239
0.00001	0.5026	0.1633	0.5429	0.4001	0.5687

As Fig. 3 indicates, both Policy II and III can improve the inspection process in terms of *PFR* under each constraint. The improvements are investigated and depicted in Table 2 as the percent reduction in *PFR* values, e.g. column 1 is

$$\frac{(\text{Policy I} - \text{Policy II})}{\text{Policy I}} \cdot 100\%.$$

Columns 2 and 3 suggest that both Policy II and III have fairly reasonable improvement over Policy I. Comparing the values in column 2 and 3 row by row, it is noted that the improvement of Policy III over Policy I is always larger than that of Policy II over Policy I. Meanwhile, the last column of Table 2 also shows the improvement of Policy III over Policy II, all of which are positive values. These results agree with our intuition that Policy III, which uses sensors with higher precision in each subsequent re-inspection, should typically perform better than the other policies in most cases depending on the choice of parameters. Policy II is in fact equivalent to a special case of Policy III. Note that the average of the $l-1$ re-inspection measurements $\bar{r}_{2:l}$ follows the distribution $\bar{r}_{2:l}|x=i \sim N(\mu_i, \sigma_i^2 + \sigma_\varepsilon^2/(l-1))$ for $i = 0, 1$. This would be the same as the measurement of the first re-inspection if the precision level of the sensor in the re-inspection was such that $\sigma_{\varepsilon,2} = \sigma_\varepsilon/\sqrt{l-1}$. Therefore, when $\sigma_{\varepsilon,2} < \sigma_\varepsilon/\sqrt{l-1}$, the first two steps of Policy III perform better than Policy II. Our numerical example is such a case.

The discussion above confirms that repeat inspection can reduce the negative effects of measurement error and repeat inspection by sensors with increased precision level can further reduce the effects of measurement error on the classification performance of inspection policies.

Table 2 Percent improvement

*FA**	*PFR*		
	Policy II vs. Policy I	Policy III vs. Policy I	Policy III vs. Policy II
0.001	19.83%	30.45%	13.24%
0.0001	20.94%	32.31%	14.38%
0.00001	21.66%	33.50%	15.12%

Table 3 Distribution parameters for numerical examples

Series system	Series-parallel system	Parameters					
sensor	(subsystem, sensor)	π	σ_0	σ_1	σ_ε	$\sigma_{\varepsilon,2}$	$\sigma_{\varepsilon,3}$
1	(1, 1)	0.0002	0.25	0.15	0.06	0.03	0.015
2	(1, 2)	0.0001	0.3	0.1	0.08	0.04	0.02
3	(2, 1)	0.0003	0.35	0.13	0.1	0.05	0.025
4	(2, 2)	0.00025	0.2	0.16	0.07	0.035	0.0175

5.2 Numerical examples for container inspection systems

This subsection presents examples of container inspection systems. The three policies presented earlier in the paper are compared for systems configured as series and series-parallel Boolean functions using four sensors for each system. Similar to the numerical example for a single station given in Sect. 5.1 and without loss of generality, we choose $\mu_0 = 0$, $\mu_1 = 1$ for all attribute distributions. We also choose some reasonable values for π and the assumed distribution parameters, provided in Table 3. Here, as in Sect. 5.1, the total number of inspections for Policy II and III is set to three. In Table 3, σ_ε presents the standard deviations of measurement errors in the initial inspection. The standard deviations $\sigma_{\varepsilon,2}$ and $\sigma_{\varepsilon,3}$ correspond to the measurement errors for the second and third inspections of Policy III. The optimization goal for each policy is to minimize the objective function *PFR*, subject to the constraint that the *PFA* value is less or equal than 10^{-3}, 10^{-4} and 10^{-5} respectively. The expressions for *PFR* and *PFA* for series and series-parallel systems can be found in Sect. 4.1 and Sect. 4.3. The optimization results reported include the optimal objective function values and the associated optimal threshold values and band widths. Note that since $\sigma_{\varepsilon,2} < \sigma_\varepsilon/\sqrt{2}$ in Table 3, Policy III is expected to perform better than Policy II.

Table 4 shows the optimization results obtained for the series inspection system. It can be seen from the table that for each inspection policy, the smaller the value of the constraint on *PFA*, the larger the achieved optimal *PFR* value is. Also, within each of the three sets of constraints, the optimal *PFR* value of Policy III is smaller than that of Policy II, and the value of Policy II is smaller than that of Policy I. Thus, for the series Boolean configuration and under the set of given parameters, the performance of Policy III is better than that of Policy II, and Policy II is better than Policy I in terms of achieved *PFR* values. In addition, the optimal threshold values of Policy III are slightly higher than that of Policy II, and the optimal threshold values of Policy II are slightly higher than that of Policy I. For Policy III, the second band width is smaller than the first band width, which is consistent with our understanding that each re-inspection band further narrows the population for subsequent inspection.

Table 4 Optimal threshold values and band width for series system with four sensors

*FA**	Policy	Sensor 1			Sensor 2			Sensor 3			Sensor 4			*PFR*
		T_1	$b_{1,1}$	$b_{1,2}$	T_2	$b_{2,1}$	$b_{2,2}$	T_3	$b_{3,1}$	$b_{3,2}$	T_4	$b_{4,1}$	$b_{4,2}$	
10^{-3}	I	.503	–	–	.616	–	–	.508	–	–	.448	–	–	0.1411
	II	.527	.149	–	.658	.218	–	.563	.266	–	.471	.181	–	0.0982
	III	.543	.321	.114	.687	.416	.150	.600	.691	.268	.486	.372	.137	0.0759
10^{-4}	I	.410	–	–	.534	–	–	.398	–	–	.357	–	–	0.2557
	II	.436	.154	–	.585	.219	–	.465	.273	–	.385	.182	–	0.1832
	III	.453	.460	.113	.620	.585	.154	.510	.943	.188	.402	.562	.139	0.1431
10^{-5}	I	.328	–	–	.465	–	–	.307	–	–	.274	–	–	0.3945
	II	.355	.157	–	.523	.226	–	.382	.281	–	.306	.184	–	0.2955
	III	.373	.448	.113	.563	.580	.156	.433	.908	.191	.326	.552	.132	0.2366

Table 5 Optimal threshold values and band width for series-parallel system with four sensors

*FA**	Policy	Sensor 1			Sensor 2			Sensor 3			Sensor 4			*PFR*
		T_1	$b_{1,1}$	$b_{1,2}$	T_2	$b_{2,1}$	$b_{2,2}$	T_3	$b_{3,1}$	$b_{3,2}$	T_4	$b_{4,1}$	$b_{4,2}$	
10^{-3}	I	.488	–	–	.577	–	–	.428	–	–	.442	–	–	.0032
	II	.511	.185	–	.628	.295	–	.490	.208	–	.469	.189	–	.0014
	III	.523	.217	.102	.663	.304	.150	.519	.249	.071	.484	.246	.109	.0008
10^{-4}	I	.384	–	–	.500	–	–	.337	–	–	.335	–	–	.0138
	II	.411	.174	–	.559	.272	–	.412	.288	–	.367	.179	–	.0063
	III	.429	.246	.105	.600	.359	.151	.463	.560	.213	.387	.371	.138	.0035
10^{-5}	I	.296	–	–	.433	–	–	.256	–	–	.241	–	–	.0407
	II	.325	.159	–	.500	.228	–	.341	.286	–	.278	.187	–	.0197
	III	.343	.260	.107	.546	.405	.152	.398	.683	.254	.302	.403	.143	.0115

Table 5 presents the optimization results obtained for the series-parallel inspection system. It displays a similar trend in terms of threshold values, band width, and minimal *PFR* values under different constraints. This again confirms that for the given parameters, Policy III performs better than Policy II, and Policy II performs better than Policy I in minimizing *PFR*.

The numerical examples presented in this section demonstrate that optimal inspection policy parameters can be obtained with the formulation of the optimization problem in single stations as well as in systems applying different Boolean decision functions. They confirm that strategies such as the use of a re-inspection band and repeat inspection with increased precision level can reduce container misclassification errors. Policy III which integrates both proposed strategies attains the maximum improvement out of the proposed policies in terms of *PFR*.

6 Conclusions

In this paper, we consider a problem often encountered in practice: container inspection at port-of-entry where measurements of inspection devices (or sensors) have errors. We develop measurement error models, formulate and investigate three inspection policies and compare their performance in terms of container misclassifications. We show that ignoring measurement errors results in greater container misclassification. This has been demonstrated using a single inspection station which is then generalized to entire inspection systems composed of several stations arranged in structured configurations such as series, parallel, series-parallel and parallel-series. The results show that measurement errors indeed have a significant effect on misclassifying containers. The consequence of such misclassification could be significant when containers with undesired cargo pass through the system without detection. Therefore, it becomes prudent that the optimization problems described in this paper include other objectives such as cost minimization (see Elsayed et al. 2009 as an example).

The inspection policies studied in this paper are not inclusive and other policies can be used to minimize the impact of measurement errors. In addition, the present work can be extended to systems with unstructured configurations such as directed networks of inspection stations.

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Appendix

If an individual sensor makes a decision on an attribute following Policy I, then

$$P(D_i=0|X_i=x_i)=\begin{cases}\Phi\left(\frac{T_i-\mu_0}{\sqrt{\sigma_{0,i}^2+\sigma_{\varepsilon,i}^2}}\right) & \text{if } x_i=0\\ \Phi\left(\frac{T_i-\mu_1}{\sqrt{\sigma_{1,i}^2+\sigma_{\varepsilon,i}^2}}\right) & \text{if } x_i=1.\end{cases}$$

If an individual sensor makes a decision on an attribute following Policy II, then

$$P(D_i=0|X_i=x_i)$$

$$=\begin{cases}\Phi\left(\frac{T_i-\frac{b_i}{2}-\mu_0}{\sqrt{\sigma_{0,i}^2+\sigma_{\varepsilon,i}^2}}\right)+\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-y_i}{\sigma_{\varepsilon,i}/\sqrt{l-1}}\right)\left\{\Phi\left(\frac{T_i+\frac{b_i}{2}-y_i}{\sigma_{\varepsilon,i}}\right)-\Phi\left(\frac{T_i-\frac{b_i}{2}-y_i}{\sigma_{\varepsilon,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}}dy_i\\ \text{if } x_i=0\\ \Phi\left(\frac{T_i-\frac{b_i}{2}-\mu_1}{\sqrt{\sigma_{1,i}^2+\sigma_{\varepsilon,i}^2}}\right)+\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-y_i}{\sigma_{\varepsilon,i}/\sqrt{l-1}}\right)\left\{\Phi\left(\frac{T_i+\frac{b_i}{2}-y_i}{\sigma_{\varepsilon,i}}\right)-\Phi\left(\frac{T_i-\frac{b_i}{2}-y_i}{\sigma_{\varepsilon,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}}dy_i\\ \text{if } x_i=1.\end{cases}$$

If an individual sensor makes a decision on a specific attribute according to Policy III, then

$$P(D_i=0|X_i=x_i)$$
$$=\begin{cases}\Phi\left(\frac{T_i-\frac{b_{i,1}}{2}-\mu_0}{\sqrt{\sigma_{0,i}^2+\sigma_{\varepsilon,1,i}^2}}\right)+\sum_{k=2}^{l-1}\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-\frac{b_{i,k}}{2}-y_i}{\sigma_{\varepsilon,k,i}}\right)\\ \quad\times\prod_{j=1}^{k-1}\left\{\Phi\left(\frac{T_i+\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)-\Phi\left(\frac{T_i-\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}}dy_i\\ \quad+\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-y_i}{\sigma_{\varepsilon,l,i}}\right)\prod_{j=1}^{l-1}\left\{\Phi\left(\frac{T_i+\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)-\Phi\left(\frac{T_i-\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_0}{\sigma_{0,i}}\right)}{\sigma_{0,i}}dy_i\\ \quad\text{if } x_i=0\\ \Phi\left(\frac{T_i-\frac{b_{i,1}}{2}-\mu_1}{\sqrt{\sigma_{1,i}^2+\sigma_{\varepsilon,1,i}^2}}\right)+\sum_{k=2}^{l-1}\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-\frac{b_{i,k}}{2}-y_i}{\sigma_{\varepsilon,k,i}}\right)\\ \quad\times\prod_{j=1}^{k-1}\left\{\Phi\left(\frac{T_i+\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)-\Phi\left(\frac{T_i-\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}}dy_i\\ \quad+\int_{-\infty}^{\infty}\Phi\left(\frac{T_i-y_i}{\sigma_{\varepsilon,l,i}}\right)\prod_{j=1}^{l-1}\left\{\Phi\left(\frac{T_i+\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)-\Phi\left(\frac{T_i-\frac{b_{i,j}}{2}-y_i}{\sigma_{\varepsilon,j,i}}\right)\right\}\frac{\phi\left(\frac{y_i-\mu_1}{\sigma_{1,i}}\right)}{\sigma_{1,i}}dy_i\\ \quad\text{if } x_i=1.\end{cases}$$

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